Moore's Law and the Semiconductor Industry:
A Vintage Model

Ana Aizcorbe
Samuel Kortum

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Dale Jorgenson's Presidential address to the American Economic Association (2001) makes a convincing case that accelerated technological change in the production of semiconductors, microprocessors in particular, has driven the recent increased productivity growth in the U.S. economy. But, while semiconductors now figure prominently in accounts of economic growth, Jorgenson points out that there is not a fully satisfactory economic model of the industry that produces them. This paper is our attempt to rise to Jorgenson's challenge.

We develop a model of the semiconductor industry and apply it to the sector producing microprocessor chips (MPU's). Our intention is to produce a model that: (i) fits the basic facts about this sector, (ii) explains the link between technological improvements, price declines, and product introductions (see Jorgenson,

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2001), and (iii) clarifies how competition in the industry influences prices and product introductions (see Aizcorbe, 2004). We put the theoretical model to work in examining how the industry responds to quickening technological change. We find, as expected, that faster technological change leads to faster declines in chip prices and shortened lives of individual chips. More surprisingly, we find that the introduction prices of chips will be higher in an environment with faster technological change. These results hold across two polar cases of market structure, perfect competition and monopoly.

Perhaps the reason others have not taken up Jorgenson's challenge is that, on its surface, the industry appears so simple. On the one hand, we observe a relationship called Moore's Law: an amazingly rapid exponential increase in the performance of chips over time (see Figure 1). On the other hand we observe a similarly rapid exponential decline in semiconductor price indices and in the prices of individual microprocessors over their product life (see Tables 1 and 2). Clearly the price declines of existing products are necessitated by the fact that they must stay competitive with newly introduced chips whose better performance traces out Moore's Law. We can make sense of these observations by treating chips as homogenous except that newer chips provide more of whatever older chips provide. There is no apparent need for a sophisticated model of product differentiation, of strategic interaction among producers, or of learning-by-doing in semiconductors, to understand these most basic relationships between technological change and prices in the industry.

Yet, on deeper inspection, other features of the industry are more puzzling. We typically observe a number of different chips, from the one just introduced to products that have been around one, two, or even three years, all on the market at the same time. Why doesn't the best product drive the others out of business? Why does the industry continue to produce an inferior chip? Our first stab at a model of the industry attempts to maintain the simplicity of treating MPU's as a commodity while coming to grips with this observation about the availability of a hierarchy of products, all on the market at any given date.

We start with a model of a competitive industry producing microprocessors. Growth in the industry is driven by Moore's Law which, in the model, reflects improvements in the technology of chip-producing equipment. Since technological improvements are embodied in the equipment for production, we are led to the model of vintage capital, developed by Salter (1960).[2] Investing in any vintage of chip-producing equipment is a sunk cost. Thus, a given product continues to be sold even when it is no longer the best performing chip on the market. It drops out of the market only when the competitive price drops below the marginal cost of production.

This straightforward augmentation of the simple competitive model takes us a long way. It delivers a convincing producer-side explanation for the rapid declines in the prices of individual microprocessors over their life on the market. Producers must make massive investments in chip producing equipment for each new chip they introduce. These investments are specific to a particular chip and are irreversible. The cost of such investments can only be recouped if the price of a chip is far above the unit variable cost of producing it when it is introduced. Producers anticipate that this markup will rapidly deteriorate, however, as new and better

[2]The vintage capital model is laid out more formally by Solow et. al. (1966). Moen and Wallerstein (1997) show how the vintage capital model can be used to address issues of inequality in the labor market.

chips are introduced. We can interpret the price declines as either declines in price markups over variable costs (given that the cost of the equipment is sunk) or as declines in costs themselves, if we include in variable cost the high but rapidly declining implicit rental cost of the equipment (the value of the equipment falls to zero when the chip drops out of the market). In either interpretation, this explanation stands in sharp contrast to the two most popular explanations for the observed price declines: (i) costs fall due to learning by doing or (ii) markups fall either due to a loss of market power as chips age and become a less differentiated commodity (Song, 2004; Hobijn, 2000) or due to intertemporal price discrimination (Aizcorbe, 2004). While these other explanations may have a role in a complete quantitative model, we feel that the simple explanation coming out of the vintage model should be the starting point.

Of course one may question the relevance of a competitive model when applied to an industry dominated by a single firm, in this case Intel. We are able to show, however, that the predictions of the competitive model carry over, in large part, to the analysis of a monopolist. In particular, the price paths of individual products are unchanged up to a time-specific factor capturing the ratio of price to marginal revenue at any date. We fit this extension of the model to data on the microprocessor industry and use the model to evaluate consequences of the speed up in Moore's Law that is thought to have occurred in the mid 1990's.

Our model builds on a number of strands of the literature. As noted above, it shares many of the features of vintage capital models. The industry equilibrium setting is borrowed from Lucas and Prescott (1972). The model most like ours is that of Jovanovic and Lach (1989). The main difference is that while they think of improvements in production equipment as being driven by learning, we take these improvements to be an exogenous function of time. We understand that our contribution is not in the analytics, but perhaps, is in our attempt to use the model to understand the behavior of the semiconductor industry.

1 The Microprocessor Sector

The semiconductor industry is credited with one of the fastest rates of product innovation and technical change within manufacturing. Chipmakers generate wave after wave of ever more powerful chips at prices comparable to those that already exist. Within semiconductors, product innovation has been especially rapid at Intel, the world's leading producer of the microprocessor chips that serve as the central processing unit in computers. Developments in the microprocessor sector appear to have been an important driver of overall productivity growth as advances in these chips paved the way for co-invention in downstream industries that, taken together, provide firms with more efficient ways to do business.

The pace of technological improvement for the semiconductor market is often referred to as Moore's Law, which states that the number of electrical components on a chip will double over a specified time period, taken to be about 24 months.[3] The likely pace of Moore's Law is studied by members of an industry-wide consortium called ITRS (International Technology Roadmap for Semiconductors). Working teams made up of chipmakers, semiconductor equipment manufacturers, and materials producers meet regularly to assess their ability to jointly push out the frontier. The resulting assessment is published once a year as the "ITRS Roadmap" and contains the expected, or most likely, path for Moore's Law out 10

[3] See Flamm (2003) for an interesting history of Moore's Law (Moore, 1965).

or so years into the future.

An important driver of this rapid rate of innovation lies in the equipment used to manufacture chips. The development of equipment capable of etching finer circuitry is often referred to as "process innovation" and the length of time between the introduction of the new equipment as the "product cycle." Beginning in 1995, the industry moved from a three-year to a two-year product cycle and, thus, opened up possibilities for an increase in the rate of product innovation.[4] This occurs because the sophistication of the lithography equipment determines the size of features on chips (transistors) so that equipment that can etch narrower circuitry can include more features on each chip and, hence, can increase the chip's quality.

For MPU chips, the increase in Moore's Law in 1995 is shown in the first line of Table 1: the growth rate for the number of transistors on Intel's chips accelerated from 24 percent in the 1985-94 period to 43 percent in 1995-99. This pickup is also reflected in an important attribute of MPUs, their speed (see line 2 of Table 1).

The prices for Intel's chips began to decline more rapidly after 1995. In a study of the sources of productivity growth, Jorgenson (2001) noted an inflection point in the constant-quality price indexes for semiconductors (line 3 of Table 1) that was generated, in large part, by a pronounced inflection point in the MPU price index (line 4). Because more rapid declines in these price measures typically reflect faster rates of measured productivity, Jorgenson and others have hypothesized that the inflection point in the price index reflects a speed-up in Moore's Law that was enabled by the shift in the product cycle. These phenomena were viewed

[4]For additional details about this shift to a shorter technology cycle, see the International Technology Roadmap for Semiconductors (2001 and 2002 update).

as important developments because the rapid price declines seen for semiconductors contribute importantly to increases in labor productivity for the economy as a whole.[5] However, the link between the product cycle and productivity measures is not yet well understood and, indeed, Jorgenson has emphasized the need for formal models of the semiconductor industry and MPU sector to better understand the role of product cycles in generating productivity gains for the overall economy.[6]

Product-level data for Intel's chips show how the inflection point in the price indices is reflected in the pricing patterns of the underlying chips. Table 2 gives summary statistics for Intel desktop chip families introduced from 1985 to 1999: the 386, 486, Pentium I, Pentium II, and Pentium III chips. The data include the introduction and exit prices, the number of years the chip was on the market, and the average annual percentage price decline over the lives of the chips.

As shown in Table 2, introduction and exit prices for chips were fairly stable over the period. The exception was the 386 which was introduce at a much lower price and which exited at a slightly lower price. Other than that, the introduction prices were in the $600-750 range while the exit prices were in the $100-150 range. The length of time individual chips spent on the market declined throughout the period. Chips introduced before 1995 were around for nearly 3 years while

[5]These issues are discussed in Oliner and Sichel (2000, 2002), Jorgenson and Stiroh (2000), and Jorgenson, Ho, and Stiroh (2002), Congressional Budget Office (2002), and McKinsey Global Institute (2001).

[6]There have been surprisingly few attempts to model the semiconductor industry. For two models of the DRAM sector see Baldwin and Krugman (1988) and Flamm(1996). There has also been work to formally model and estimate the demand for these devices (see, Song (2003)) as well as the cost parameters facing chip makers (see, Irwin and Klenow (1994) and Siebert (2002)).

those introduced after 1995 lasted less than 2 years.

Data for the individual chips that make up these families, available since 1993, provide a closer look at the pricing patterns. As seen in the top panel of Figure 2, MPU prices for each chip start at between \$600 to \$1000 at introduction and fall steadily until the chip exits the market, by which point its price has typically fallen to around \$100. The price contours become very regular starting with the chips introduced after 1997. Our interest in developing a stylized model of the industry was in part motivated by this striking pattern.

Early studies of the semiconductor industry found similar contours for prices for memory chips and attributed the downward-sloping nature of price contours to learning economies.[7] While learning may, indeed, be an important determinant of prices for memory chips, the evidence for MPU prices is more mixed. Hobijn (2001) has argued that, given Intel's dominance of the MPU market, declines in the prices of individual chips are more likely to reflect falling markups of price over cost rather than falling costs. Using estimates derived from an econometric model of demand for MPUs, Song (2001) estimated that markups were substantial and fall over the life of each MPU chip. Similarly, using industry estimates for Intel's costs, Aizcorbe (2002) argued that costs are so low relative to price (less than \$100) that even substantial decline in costs would not pull down prices

[7]See Hatch and Mowery (1998) for discussion of the sources of learning curves in semiconductor production as well as a recent review of empirical studies devoted to estimating these learning curves. Because the needed cost data are not readily available, empirical studies of the learning curves typically use prices as a proxy for cost. The two exceptions are Irwin and Klenow (1994) and Siebert (2002), where structural models that specify the relationship between price and marginal cost are used to obtain learning curve estimates, and Hatch and Mowery (1998), where a unique survey was used to obtain the needed data.

sufficiently to generate these contours. Finally, declining markups over the life of each chip may also reflect intertemporal price discrimination over heterogeneous buyers (Aizcorbe, 2004). Our model will deliver a very different version of this declining markup explanation, one that holds even under perfect competition.

The graph shows that price contours for Intel's chips became steeper in the late 1990s. This steepening is most easily seen by comparing the pre-1995 contours with those after 1997 in Figure 2. The figure confirms what is seen at a more aggregate level in Table 2; prices dropped more rapidly after 1995 at the same time that chips' lifespans shrank. To summarize, there were two changes in the price contours for individual MPU chips that coincided with the increase in Moore's Law: price contours became steeper and the length of time chips stayed on the market decreased.

In what follows, we develop a stylized theoretical model that delivers the regular pattern in price contours seen in Figure 2. We then analyze whether the model can explain the changes in pricing that we observe, based on an acceleration in Moore's Law.

2 A Competitive Industry

Our baseline model is a competitive industry with many chip producers. Each producer may make chips of a different quality. A chip producer takes the price it gets for its chips as given. Time is continuous. Producers discount future profits at rate r.

2.1 Demand

We model the demand for chips in the simplest possible way. Consumers care only about quality units. They are indifferent between having one chip of quality A or A chips of quality 1. The output of the industry can therefore be summarized by the quantity of quality-weighted chips, X. The associated price per quality unit of chip is P.

We posit a simple demand curve giving the price at which a quantity X would be demanded:

$$P = D(X; t).$$

At any date t the function $D(X; t)$ is downward sloping, $D'(X; t) < 0$. Indexing the function by t allows for possible shifts in demand. For some purposes it is useful to denote the quantity demanded X^D as

$$X^D(P; t) = D^{-1}(P; t). \tag{1}$$

2.2 Supply

The supply side is more intricate. Suppose a producer enters the industry at date t by building a fabrication plant of physical capacity I (chips per unit time). The the total sunk cost of building such a plant is qI, where q is the cost per unit of capacity. When the plant is running, the variable cost is a constant c per chip. Thus if a plant of size I produces chips at full capacity for l years: (i) total production over the life of the plant is Il, (ii) total variable cost is Ilc, and (iii) total cost, including the sunk cost of building the plant, is $(q + lc)I$.

We assume that the cost of building the plant (or retrofitting it) is the cost of the chip producing equipment. A plant built at date t embodies equipment of vintage

$v = t$. Such a plant produces chips of quality $A(v)$. The notation v captures the fact that once the plant is built, the producer has locked in a particular vintage of technology and the plant itself will experience no technological change. Over time, however, new and better vintages of equipment become available, that is $A'(v) > 0$. Looking back in time, it is convenient to assume that $\lim_{v \to -\infty} A(v) = 0$.

When we want to parameterize the model we will assume $A(v) = e^{gv}$. The parameter g is the rate of technological change. According to Moore's Law $g = 0.35$. We assume that producers anticipate the working of Moore's law, or more generally anticipate the evolution of $A(v)$. We take the path of $A(v)$ as exogenous to the producers in our model.[8]

We denote cumulative investment in the industry, up to and including date t, by $K(t)$. Thus, for $v \leq t$, $K(v)$ is the total capacity of all equipment of vintage v or earlier, and $K(t) - K(v)$ is the total capacity of equipment strictly more advanced than vintage v. The constant returns to scale technology that we have assumed allows us to ignore investments in individual plants and to simply keep track of their sum. We keep track of the stock of equipment rather than the flow to accommodate the potential for spikes of investment at particular dates. Hence $K(v)$ need not be differentiable. Since investments are sunk, $K(t)$ is non-decreasing.

The output of the industry is simply quality-weighted chips. Thus we can aggregate across vintages after weighting the physical output of vintage v chips

[8]One could make technological progress endogenous via learning by doing. Irwin and Klenow (1994) estimate significant *learning by doing* in memory chips. Jovanovic and Lach (1989) show how to incorporate A increasing with cumulative industry investment. We do not find it plausible, however, that learning by doing is the main force behind the slope of Moore's Law for microprocessors. We leave it for future work to explain what actually drives Moore's Law.

by $A(v)$. At any date t, we only need to keep track of a single price $P(t)$, the price of a quality unit. There is overwhelming evidence (see Table 1) that price per quality unit has been falling rapidly over time. It will simplify the discussion, prior to imposing the conditions for industry equilibrium, to simply require that $P(t)$ be non-increasing in t.

A fabrication plant built at date t, hence embodying equipment of vintage $v = t$, will remain operative only for some period of time, $l(v)$. The reason the plant shuts down is that otherwise it would lose money. The plant operates only as long as its revenue per physical unit exceeds its unit production cost c:

$$\begin{aligned} P(s)A(v) \quad &\geq \quad c \quad \text{for} \quad v \leq s \leq v + l(v) \\ &< \quad c \quad \text{for} \quad s > v + l(v). \end{aligned} \tag{2}$$

We thus obtain a **shutdown condition**:

$$P(v + l(v))A(v) = c, \tag{3}$$

or else, if no finite $l(v)$ solves this equation, we set $l(v) = \infty$, noting that vintage v will then be operated forever.

A second condition, in this case involving the sunk cost per unit of capacity, is that entrants take advantage of all profit opportunities for investing in the industry. In other words, the returns to investing in a unit of capacity of vintage v can never exceed the cost of doing so. This **investment condition** is

$$q \geq \int_{v}^{v+l(v)} e^{-r(s-v)}[A(v)P(s) - c]ds, \tag{4}$$

which holds with equality if there is any investment in vintage v equipment. The investment condition says that the sunk cost per unit of capacity must exceed the discounted net cash flow per physical unit produced, given that a plant built at date t will continue to operate until date $t + l(t)$. As with the shutdown condition, the investment condition holds for each vintage $v = t$.

With plants of vintage v operating for $l(v)$ periods, at date t the output of the industry will include all vintages $v \leq t$, such that $v + l(v) \geq t$. To simplify notation we define $\tau(t)$ to be the age of the oldest plant still operating at date t. The shutdown condition implies $P(t)A(t - \tau(t)) = c$, and hence $l(t - \tau(t)) = \tau(t)$.

Integrating over past vintages, the total flow of quality units that can be produced at date t, using vintages $v \in [t - \tau(t), t]$, is $\int_{t-\tau(t)}^{t} A(v)dK(v)$. An exogenous reduction in the industry price at some date t would cause older vintages to halt production leading to a reduction in X. To see this formally, note that the lower endpoint of integration is $t - \tau = A^{-1}(c/P)$, which is decreasing in P. It is thus convenient to write the industry supply curve as

$$X^S(P;t) = \int_{A^{-1}(c/P)}^{t} A(v)dK(v). \tag{5}$$

The supply curve is typically smoothly increasing in P. It will jump at a price P, however, if there is a mass of investment in vintage $v = A^{-1}(c/P)$. In this case $X^S(P;t)$ is an upper bound on supply since it entails vintage $v = A^{-1}(c/P)$ being operated at full capacity.

2.3 Equilibrium

At any date t, taking account of any new investment at that moment, the equilibrium price per quality unit $P(t)$ must induce a supply of quality units of chips (5) that is sufficient to match demand (1) at that price. Taking account of the possibility that the vintage $v = A^{-1}(\frac{c}{P(t)})$ may be only partially utilized, we get the

market-clearing condition:

$$P(t) = \min \left\{ P : X^S(P;t) \geq X^D(P;t) \right\}. \tag{6}$$

Since the market clearing condition involves the price at date t, it is convenient to express the shutdown condition in terms of the date t price. To do so, we consider the oldest active vintage, $v = t - \tau(t)$, so that $v + l(v) = t - \tau(t) + l(t - \tau(t)) = t$. Similarly, the investment condition can be expressed in terms of the condition for investment in vintage $v = t - \tau(t)$.

The initial condition for the industry, as of date t_0, is the capacity profile $K(v)$ over vintages $v < t_0$. We have now defined the shutdown condition, the investment condition, the market-clearing condition, and the initial condition. Using these conditions, we are prepared to define a competitive equilibrium for the industry.

The *industry competitive equilibrium* is a set of time paths, over all dates $t \geq t_0$, of prices $P(t)$ (non-increasing in t), age of the oldest productive vintage $\tau(t)$, and total capacity $K(t)$ (non-decreasing in t) such that:

1. The initial condition is given by $K(v)$, for all $v < t_0$.

2. The shutdown condition, given by (3), holds for all $v = t - \tau(t)$ such that $t \geq t_0$.

3. The investment condition, given by (4), holds for all $v = t - \tau(t)$ such that $t - \tau(t) \geq t_0$.

4. The market-clearing condition, given by (6), holds for all t.

We do not provide the conditions under which such an industry equilibrium exists. Such conditions will be easier to interpret in the more restrictive setting of a stationary industry equilibrium, which we consider later. In the meantime, we examine how our analysis can be extended to cover the case of monopoly.

3 Monopoly

The analysis above assumes competitive price-taking behavior in the industry. Here we go to the opposite extreme and consider a monopoly. We assume that the monopolist is interested in maximizing the discounted value of industry revenue less the cost of investment and production.

The industry revenue function is

$$R(X;t) = XD(X;t). \tag{7}$$

Marginal revenue at date t, denoted $M(t)$, is

$$M(t) = R'(X(t);t) = X(t)D'(X(t),t) + D(X(t),t).$$

To guarantee that the monopolist's problem is bounded and well behaved, we assume that marginal revenue is strictly decreasing in X. A sufficient condition is that $D(X,t)$ is weakly concave in X. Some convex demand curves, for example a constant elasticity demand curve with an elasticity of demand strictly greater than one, also yield marginal revenue decreasing in X.

The analysis of monopoly is much like the analysis of the competitive industry except that marginal revenue per quality unit $M(t)$, rather than price per quality unit $P(t)$, guides the production and investment decisions of the monopolist. To see this connection, it is convenient to have an expression for the level of demand consistent with any given level of marginal revenue,

$$X^{D_M}(M;t) = R'^{-1}(M;t) \tag{8}$$

But, our observations on the industry are prices not marginal revenues. To map from one to the other we define:

$$\mu(t) = \frac{P(t)}{M(t)} = \frac{\varepsilon(t)}{\varepsilon(t) - 1},$$

where $\varepsilon(t)$ is the elasticity of demand. To be more explicit about the sources of variation in this ratio, we can write $\mu(t) = \mu(X(t);t)$ where $\mu(X;t) = D(X;t)/R'(X;t)$. Thus $\mu(t)$ will vary with time due both to changes in output as well as due to shocks to demand.

As in the competitive equilibrium, there are three conditions that characterize industry behavior when the industry is monopolized: the monopoly shutdown condition, the monopoly investment condition, and the monopoly market-clearing condition. We consider these conditions in turn.

The monopolist will stop using a plant when the marginal revenue it generates falls below the unit cost of production. Thus, the monopolist will operate vintage v from date v to date $v+ l(v)$ and will then shut it down, where:

$$\begin{aligned} M(s)A(v) &\geq c \quad \text{for} \quad v \leq s \leq v + l(v) \\ &< c \quad \text{for} \quad s > v + l(v), \end{aligned} \tag{9}$$

The **monopoly shutdown condition** is thus:

$$M(v + l(v)) = c/A(v). \tag{10}$$

or else $l(v) = \infty$. Note that a monopolist will shut down a plant well before the date at which revenue from the plant falls below the operating cost. The monopolist restricts industry output, keeping the industry price above the competitive level, by not producing anything with older vintages of technology. In short, the monopoly shutdown condition is simply the familiar marginal revenue equals marginal cost condition as it applies to the worst plant operated by the monopolist.

To motivate the investment condition, it is helpful to reinterpret it as a condition for efficient production. Consider some plan for the production of efficiency units over time. Given that plan, what is the implication for production costs of investing in an extra unit of vintage v equipment? At some date $s \in [v, v + l(v)]$, the extra unit of capacity in vintage v equipment permits shutting down $A(v)/A(s - \tau(s))$ units of the oldest vintage still being used at date s, with no effect on the number of quality units produced. The net savings in variable costs of shifting to more modern equipment is $cA(v)/A(s - \tau(s))A(v) - c$. Over the productive life of the unit of vintage v equipment the present value of savings $S(v)$ is

$$S(v) = \int_{v}^{v+l(v)} e^{-r(s-v)} c[A(v)/A(s - \tau(s)) - 1]ds. \tag{11}$$

Since the cost of such an investment is q, we should see no investment if $q > S(v)$. Furthermore, we should never observe $q < S(v)$ since such a situation would represent an unexploited profit opportunity. Thus, if there is any investment in vintage v equipment, we have the condition $q = S(v)$. Substituting in the competitive shutdown condition yields the investment condition (4).

A monopolist also wants to produce efficiently. Hence, if the monopolist invests in vintage v equipment it follows that $q = S(v)$. Substituting the monopoly shutdown condition into (11) yields the **monopoly investment condition**:

$$q \geq \int_{v}^{v+l(v)} e^{-r(s-v)} [A(v)M(s) - c]ds, \tag{12}$$

which holds with equality if there is any investment in vintage v equipment. Another way to look at the monopoly investment condition is by comparison with the investment condition for the competitive industry. The only difference is that the monopolist values output at marginal revenue instead of price.

We can rewrite the monopoly shutdown condition as a condition for the oldest vintage $v = t - \tau(t)$ still being operated at date t. This oldest vintage in use at date t satisfies the equation $M(t)A(t - \tau(t)) = c$. The quantity X that the monopolist will be willing to supply when marginal revenue per quality unit is M is

$$X^{S_M}(M;t) = \int_{A^{-1}(c/M)}^{t} A(v)dK(v). \tag{13}$$

The **monopoly market-clearing condition** is therefore:

$$M(t) = \min\left\{M : X^{S_M}(M;t) \geq X^{D_M}(M;t)\right\}. \tag{14}$$

The market-clearing condition simply assures that the marginal revenue on which the monopolist bases production and investment decisions is in fact consistent with demand.

We now turn to a special case of the model in which some of the endogenous variables are stationary. In this special case we can sharply characterize the solution to the monopolists problem.

4 The Stationary Case

A stationary configuration of the industry is one in which technology grows at a constant rate and in which each vintage is used for a fixed period of time. By restricting our analysis to this stationary case, we can characterize the solution in much more detail. We will focus on monopoly since it is trivial to translate from the monopoly solution to the competitive solution.

We assume that the path of technology follows Moore's Law with a constant coefficient $g > 0$:

$$A(t) = e^{gt}.$$

Furthermore, in our stationary case we require that the life of a vintage is constant:

$$l(v) = \tau(t) = \tau.$$

The stationary equilibrium does not require a constant rate of investment. But, to deliver a constant τ, investment must be strictly positive at each date. Hence the monopoly investment condition holds with equality.

4.1 Prices

We get predictions about prices that hold even in settings where we can say little about investment and output. In particular, we do not need to assume any particular form for the demand curve.

Imposing the stationarity condition in the monopolist shutdown condition (10) gives $M(t + \tau) = ce^{-gt}$. It follows that the path of marginal revenue must be of the form

$$M(t) = be^{-gt}, \tag{15}$$

where b is a constant. The value of b depends on the productive life of a vintage,

$$b = ce^{g\tau}. \tag{16}$$

The monopoly investment condition (12) reduces to

$$q = \int_t^{t+\tau} e^{-r(s-t)}[be^{-g(s-t)} - c]ds.$$

Substituting in the expression for b and solving the integral,

$$q = \frac{c}{r+g}(e^{g\tau} - e^{-r\tau}) - \frac{c}{r}(1 - e^{-r\tau}). \tag{17}$$

The lifespan of a vintage, τ, is determined as the solution to (17). Given τ, the value of b is obtained from (16).

A central question in our analysis is how prices are determined by the speed of Moore's Law, g. Thus, we want to see how b and τ depend on g. To see this dependence in the simplest way, rewrite the monopoly investment condition as $q = \int_0^\tau e^{-rv}[e^{\ln b - gv} - c]dv$, where the shutdown condition $b = ce^{g\tau}$ guarantees that the integrand is never negative for $v \in [0, \tau]$. We denote this integral by $f(\tau, b, g)$. Now, suppose Moore's Law is $g' > g$. We want to find the values b' and τ' such that $b' = ce^{g'\tau'}$ and $q = f(\tau', b', g')$. If $\tau' \geq \tau$, in which case $b' > b$, we can see by inspection that $f(\tau', b', g') > q$, since $\ln b' - g'v > \ln b - gv$ for all $v \in [0, \tau]$. Thus, it must be that $\tau' < \tau$. Similarly, if $b' \leq b$, in which case $\tau' < \tau$, we can see that $f(\tau', b', g') < q$, since $\ln b' - g'v < \ln b - gv$ for all $v \in [0, \tau']$. Thus, it must be that $b' > b$. In summary, a higher g will be associated with a higher b and a lower τ. With higher g, each vintage will have a shorter life, and thus to recoup the sunk cost of investment, the initial marginal revenue must be higher.

We have pinned down the monopolist's path of marginal revenue per quality unit. It is interesting to note that we could do so without reference to the demand side of the model. The implications for the monopolist's price path is

$$P(t) = \mu(t)M(t) = c\mu(t)e^{-g(t-\tau)}. \tag{18}$$

Setting $\mu(t) = 1$ yields the price path for a competitive industry.

To verify our proposed steady state equilibrium we need to show that there is an increasing path of capacity $K(t)$ such that supply equals demand (14) given the proposed exponentially path for marginal revenue. A sufficient condition is that $X(t)$ is increasing. Thus, we must restrict the demand curve to guarantee that demand is increasing over time, i.e. $R'^{-1}(ce^{-g(t-\tau)}; t)$ is non-decreasing in t. This condition would hold automatically if we ignored demand shocks, i.e. if $D(X; t) = D(X)$. The content of the restriction is to rule out large negative shocks to demand. Our proposed steady state equilibrium would be broken by demand shocks so negative that vintages less than τ years old are taken out of production. Ruling out such large negative shocks, the steady state equilibrium has the property that positive shocks to demand as well as small negative shocks are all met by variation in new investment. It is for this reason that in our notation for the stock of capacity, we allowed for spikes in investment.

4.2 Investment and Output

Thus far we have said nothing about the implication of the model for investment in the industry, other than to require that investment be positive in the stationary case considered here. In general these implications are quite intricate. On the one hand a positive demand shock may lead to a spike of investment. But, when that spike is retired τ years later, there will be another spike to replace it. Thus in principle the path of investment reflects current shocks to demand together with echoes of past shocks. Working out these implications is not the point of this paper. We see it as an advantage of our analysis that it has strong implications for price contours without making strong assumptions about demand, and hence admitting a wide variety of behaviors for investment and output.

Here, we analyze the behavior of investment and output in a much more restrictive setting. In particular we now assume that the demand curve reflects a constant price elasticity $\varepsilon > 1$ and a secular trend λ (which may be positive or negative):

$$P(t) = D(X(t); t) = e^{\lambda t} X(t)^{-1/\varepsilon}.$$

This assumption on demand yields a number of simplifications. First, the monopoly markup is a constant $\mu(t) = \mu = \varepsilon/(\varepsilon - 1)$. Second, the level of demand consistent with marginal revenue of M becomes $X^{D_M}(M; t) = e^{\varepsilon\lambda t}(\mu M)^{-\varepsilon}$.

We conjecture, and then verify, that in this setting $K(t) = (k/h)e^{ht}$, where k and h are constants to be determined. Given the conjectured investment path, the quantity of output supplied is $X^{S_M} = \int_{t-\tau}^{t} e^{gv} k e^{hv} dv = \frac{k}{h+g} e^{(g+h)t}(1 - e^{-(g+h)\tau})$. Setting $M(t) = be^{-gt}$, applying the market clearing condition (14), and rearranging:

$$\mu b e^{-gt} = e^{\lambda t} e^{\frac{-(h+g)t}{\varepsilon}} [\frac{k}{h+g}(1 - e^{-(h+g)\tau})]^{-\frac{1}{\varepsilon}}$$

Equating the growth rates on both sides of the equation above we get $h = \varepsilon(\lambda + g) - g$. Equating the multiplicative factors on both sides, $k = \varepsilon(\lambda + g)(\mu c)^{-\varepsilon} e^{-\varepsilon g\tau}/(1 - e^{-\varepsilon(\lambda+g)\tau})$. Our conjecture is thus verified.

The equilibrium path of investment is

$$\dot{K}(t) = \frac{\varepsilon(\lambda + g)(\mu c e^{g\tau})^{-\varepsilon}}{1 - e^{-\varepsilon(\lambda+g)\tau}} e^{[\varepsilon(\lambda+g)-g]t}.$$

The path of industry output is therefore

$$X(t) = (\mu c e^{g\tau})^{-\varepsilon} e^{\varepsilon(\lambda+g)t}.$$

While output and investment are trending over time, the ratio of investment expenditure to industry revenue is constant

$$\frac{q\dot{K}(t)}{P(t)X(t)} = \frac{q\varepsilon(\lambda+g)}{\mu c e^{g\tau}(1-e^{-\varepsilon(\lambda+g)\tau})}.$$

In summary, this simplest version of the complete model has six exogenous parameters $\{q, c, g, r, \lambda, \varepsilon\}$. These parameters determine five endogenous terms $\{b, \tau, h, k, \mu\}$. Together the parameters and endogenous terms pin down the paths of price contours, investment, and output.

5 Microprocessor Prices

The sharpest and most robust results of the model relate to its implications for the price contours of individual products. The data on these price contours is shown in Figure 2. Here we focus on how the model fares in explaining these patterns. We then go on to consider what the model has to say about changes that would result from a speed up in Moore's Law. Except for such a one-time unexpected change, we will continue to impose the restriction of a stationary setting. Throughout most of the analysis, however, we can drop the assumption of a constant elasticity demand curve that we imposed in our analysis of investment and output above.

5.1 Price Contours

What are the implication of the model for how the prices of individual chips evolve over time? In relating the model to the data, it is advantageous to derive implications for the prices of particular chips rather than working with the theoretical concept of price per quality unit. We have data on the prices at various dates t of particular MPU's, defined by their introduction dates. Using the introduction date, we associate a chip with some vintage $v \leq t$. We denote by $p(t, v)$ the price at date t of the chip introduced at date v. These prices may be deflated by an aggregate price index to remove any influence of aggregate inflation.

We denote the associated marginal revenue by $m(t, v) = M(t)A(v)$. Holding fixed any vintage v, $m(t, v)$ declines at rate g with t. Vintage v drops out of the market at date $t = v + \tau$ at which point $m(v + \tau, v) = c$. The marginal revenue of any vintage of chip when it first enters the market is $m(v, v) = b$. To summarize, the marginal revenue of an MPU declines at rate g from the level b to the level c during the life of vintage v, which runs from date v to $v + \tau$.

We can take this prediction about marginal revenue to derive implications for monopoly prices of individual chips, $p(t, v) = \mu(t)m(t, v)$. Thus:

$$p(t, v) = \mu(t)be^{-g(t-v)},$$

for $0 \leq t - v \leq \tau$. In logarithms we have $\ln p(t, v) = \ln \mu(t) + \ln b - gt + gv$. Price contours will typically decline in parallel at rate g although this rate of decline may vary if the markup changes over time due to shifts in demand or movements along a demand curve that is not constant elasticity. At any given date, the more recent vintages (larger v) sell for more. But holding fixed the age of the vintage, $t - v$, the price varies only due to variation in $\mu(t)$.

Now consider what happens if the speed of Moore's Law is $g' > g$. In this case price contours become steeper (ignoring changes in $\mu(t)$). More surprisingly, the price at introduction has to be higher. Thus, with faster technological change prices fall at a faster rate but from a higher level. Products drop out of the market sooner as well.

To focus on what underlies the price declines of individual products, we temporarily fix $\mu(t) = 1$, as would be the case in a competitive industry. An important feature of our model, distinguishing it from much of the previous literature, is the mechanism driving these price declines. It is typically assumed that the price declines of individual vintages reflect falling production costs, say, due to learning by doing. Here unit production costs are fixed. What happens in our model is that markups over unit production costs fall over time. This prediction of falling markups may seem odd since they occur even under competition. But, even in a competitive environment producers need to cover their investments in equipment. The way they do so is by entering when they anticipate being able to sell their product at a price far above the unit production cost. This markup then fades away over time until eventually the product is dropped when the producer can no longer cover the unit production cost. The present value of these markups exactly cover the sunk cost of investment in chip producing machinery.

Of course what is being called a markup under perfect competition is in fact the normal return on investments in equipment. It is easy to show that with a rental market in equipment, the equilibrium rental price at each moment would absorb all the difference between the price of an MPU and the unit cost of producing it. Under monopoly, there is an additional markup of prices $\mu(t)$.

5.2 Transition Dynamics

What happens if there is a permanent unexpected increase in the speed of Moore's Law from g to g' at date t_0? Suppose that the price per quality unit at t_0 is that determined in the stationary configuration. It will simplify the discussion if we focus on the constant elasticity demand case, setting the drift in demand to $\lambda = 0$. Thus, the price level is $P(t_0) = \mu b e^{-g t_0}$ at the time when Moore's Law speeds up.

From our earlier discussion, we know that if the industry were to jump to a new stationary configuration, the price level would have to jump to $b'e^{-gt_0}$ with $b' > b$. Such a jump in price is inconsistent with market clearing. In fact, market clearing demands that there be no change in price for a period of time until date t_1, where $b'/b = e^{g'(t_1 - t_0)}$. Starting at date $t_1$the industry falls back into a stationary configuration with prices falling at rate g'.

In the interval of time from date t_0 to date t_1 industry investment falls to zero. The reason is that during this period the price of chips is not high enough to compensate investors for the higher rate of depreciation they will experience with technological change occurring at rate g'. Since there is no investment during this time interval, each chip on the market at date t_0 remains on the market at least through date t_1 and the price of each chip remains constant. After date t_1 all chip prices begin to decline at rate g' and older vintages begin to drop out of the market again. In effect, producers wait to invest until the technology of the new equipment improves enough to allow them to recoup their investments.

5.3 Numerical Illustrations

We now turn to a quantitative assessment of the model. The first part of this assessment is evaluating how well the model fits the data in the years just prior to 1995. The second part is evaluating how well the model accounts for changes that occurred between the period prior to 1995 and the period after that. Our particular focus is on consequences of the speed up in Moore's Law, which seems to have occurred around 1995.

We use the restricted form of the complete model in which there is a constant elasticity of demand. Thus, we need values for six parameters: r, λ, ε, g, c, and q. The values we have chosen are summarized in Table 3. We set the real interest rate to $r = .07$, a standard figure for the real return on equity. Our value for demand growth to $\lambda = .10$. We take the demand elasticity to be slightly below Flamm's (1999) estimate of 1.5, setting $\varepsilon = 1.3$ (his estimate applied to all semiconductor devices, not just MPU's). This elasticity implies $\mu = 4.33$. We set $g = .24$, the growth rate of Moore's Law around 1993. On the cost side, the value of c is taken from Gwennap and Tomsen (1998) who report an average production cost of \$75 per chip for the pre-1995 Pentium I chips. Our estimate of setup costs q is indirect. We use data reported in financial statements on the value of additions to machine and structures relative to total revenues, about 20 percent in the first half of the 1990s. Using the model equations to form this ratio, the value of q should satisfy:

$$q = .2\frac{\mu b(1 - e^{-\varepsilon(\lambda+g)})}{\varepsilon(\lambda + g)}.$$

This equation together with (17) and (16) can be solved jointly for q, τ, and b. By this procedure, we obtain our value for the setup cost of $q = 486$.

The first column of Table 4 shows the relevant statistics for the period following the introduction of the Pentium I chip in 1993. The second column shows what our model predicts for these statistics. As can be seen, the model implies initial prices that are reasonably close to those seen in the data but the simulated exit prices and lifespans for chips are a bit higher. The model misses because actual price declines over the life of a chip are typically much greater than measures of technological progress traced out by Moore's Law (we return to this issue below).

What does the model predict will change in response to a speedup in Moore's Law to $g' = .58$? In doing this experiment, we hold the other five parameters fixed at their baseline values given in Table 3. As seen by comparing the second and third columns, an increase in Moore's Law implies higher introduction prices (\$1071 vs. \$775), shorter lifespans (2.1 vs. 3.8 years) and faster price declines (-

58 percent vs. –24 percent). The latter two predictions are consistent with what is seen in the data following 1995; contours became steeper and chips' market lives became shorter. The prediction that introduction prices increase is not borne out; actual introduction price edged down beginning in 1995. Even so, one could still rationalize the data if at the same time that Moore's Law increased, some other parameter also changed in such a way as to hold down introduction prices. One such possibility is a decline in set up costs. The model predicts that a drop in q generates lower introduction prices (they don't need to be so high now that the set up costs are lower) and shorter lifespans, without affecting exit prices or the slope of the price contours. Another possibility would be a decline in the price elasticity, but that would affect both introduction and exit prices, and the latter did not change appreciably.

The transition to the new steady state is illustrated in Figure 3. Facing more rapidly falling prices, the firm now requires a higher introduction price for entry. Because that higher introduction price is not consistent with market clearing, there is a stall in investment until the quality units per chip, and thus the price per chip, rise enough to cover setup costs over the life of a chip. Absent entry, each chip's price remains at its t_0. At t_1 the needed introduction price becomes consistent with market clearing and entry occurs. Beyond that point, prices for all chips fall at the faster rate, $g' = .58$.

As noted above, an obvious challenge for the model is why price contours are typically much steeper than the coefficient of Moore's Law. One way to finesse that issue is to simply assume that the true rate of technical progress advances at the same rate as prices of individual products decline. Based on this assumption we set $g = .74$ for the pre-1995 period. The price contours implied by the model (column 4 of Table 4) are now much closer to what is seen in the data (column 1). Predicted exit prices still appear a bit high but the simulated introduction prices and lifespans are very close to the actual values. By construction, the rate of price declines in this case match those in the data. Moving to an even higher value of $g = .90$ for the post-1995 period, the model still predicts a substantial increase in introduction prices, something that was not seen in the data after 1995.

6 Conclusion

This paper develops a theoretical model to better understand the behavior of the microprocessor sector, an important segment within the semiconductor industry. Despite its simplicity, the model captures two important features of the MPU market.

First, the model accommodates the fact that MPUs of different qualities coexist in the market by appealing to a vintage capital framework. Producers make product-specific irreversible investments in equipment each time they introduce a new chip. Once these sunk costs have been borne, it makes sense to keep producing a chip, even after better versions have become available, as long as the price exceeds the variable cost of production.

Second, the model predicts that prices fall over the life of each chip, reflecting declines in markups for existing chips when new and superior products are introduced. The model's explanation for these price declines is a departure from the existing literature. Downward-sloping price contours are typically explained either by falling costs arising from learning economies or falling price markups arising from market-power considerations. In our model, there is no learning by doing and, although we can accommodated market power, the prediction of declining markups holds even under perfect competition. Including the implicit rental cost of equipment as part of variable cost, the falling markup becomes instead falling cost. The force driving down costs of existing products is the rapidly declining rental price on vintage-specific equipment, who's value hits zero when a product drops out of the market.

We put the model to work to get insight into the consequences of a speed up in Moore's Law. The model predicts that an increase in the coefficient of Moore's Law will: (1) increase the rate at which prices fall over the life of each chip and, thus, generate an inflection point, (2) shorten the amount of time each chip is sold on the market, and (3) increase introduction prices for each chip. Chip-level data are consistent with the first two of these predictions but do not show a noticeable increase in introduction prices. Therefore, the only way the model can rationalize the inflection point is if some other parameter changed so as to hold down introduction prices in the new steady state. One possibility, consistent with anecdotal reports that investment in plant and equipment at Intel declined in the mid-1990s, is that there was a drop in setup costs. In the model, that drop in setup costs is consistent with lower introduction prices and shorter lifespans. These predictions hold in both the competitive and monopoly versions of the model and, thus, do not require the firm to hold market power.

The most basic shortcoming of the model, in its present form, is its prediction that price declines over the life of a chip should equal the rate of technological progress. If we follow Moore's Law and measure technological progress by increases in transistors per chip or chip speed, we consistently get numbers that are substantially below the rate at which prices of individual chips decline. What accounts for this deviation between the model and the data? Our current thinking is that the problem stems from our simplifying assumptions on the demand side. We may need to consider a model in which different types of consumers buy chips on different ends of the quality spectrum. Luttmer (2004) shows how consumer heterogeneity can be incorporated into a model such as ours. With this heterogeneity, it is possible that the high-end chips would support even higher markups, leading to an additional force for rapid declines in chip prices as chips age. Even if such an extension is needed before taking the model's quantitative implications seriously, we think the present model provides a good theoretical benchmark.

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Table 1. Selected Attributes for Intel's Desktop Microprocessors and Price Indexes for Semiconductor Devices, 1985-1999

	1985-1994	1995-1999
Intel's Microprocessors		
1. Number of Transistors	24	43
2. Speed (megahertz)	17	34
Price Indexes		
3. Semiconductor Devices	- 15	- 49
4. Microprocessors	- 28	- 92

Sources: Authors' calculations based on source data on characteristics for Intel's chips from www.intel.com/pressroom/kits/quickreffam.htm, price indexes for semiconductor devices from Oliner and Sichel (2000) and indexes for microprocessors from Grimm(1999).

Table 2. Intel's Desktop Chip Families, 1985-1999

	Chip Families				
	386	486	Pentium I	Pentium II	Pentium III
1. Introduction Date	1985	1989	1993	1997	1999
2. Number of Chips	10	12	15	20	9
	----------------averages over all chips in chip family----------------				
3. Prices (dollars)					
4. Introduction	268	656	753	739	608
5. Exit	90	102	119	153	154
6. Lifespan (years)	3.00	2.75	2.50	1.75	1.50
	---------------------compound annual growth rates---------------------				
7. Price Change Over Lifespan	- 36	- 68	- 74	- 90	- 91
Growth from previous chip family:					
8. Number of Transistors	27	36	24	22	58
9. Speed of chip (megahertz)	33	11	22	34	38

Source: Introduction dates, the number of chips for each chip family, the number of transistors and introduction speed for each chip family were obtained from Intel's web site at www.intel.com/pressroom/kits/quickreffam.htm . The average prices and lifespans for chips in each family (lines 4-6) were calculated using chip-level data from Dataquest, Inc. (1985-1993) and MicroDesign Resources (1993-1999). The average price decline in line 7 is calculated by applying the formula ln(exit price/introduction price)/average lifespan to the data reported in lines 4-6.

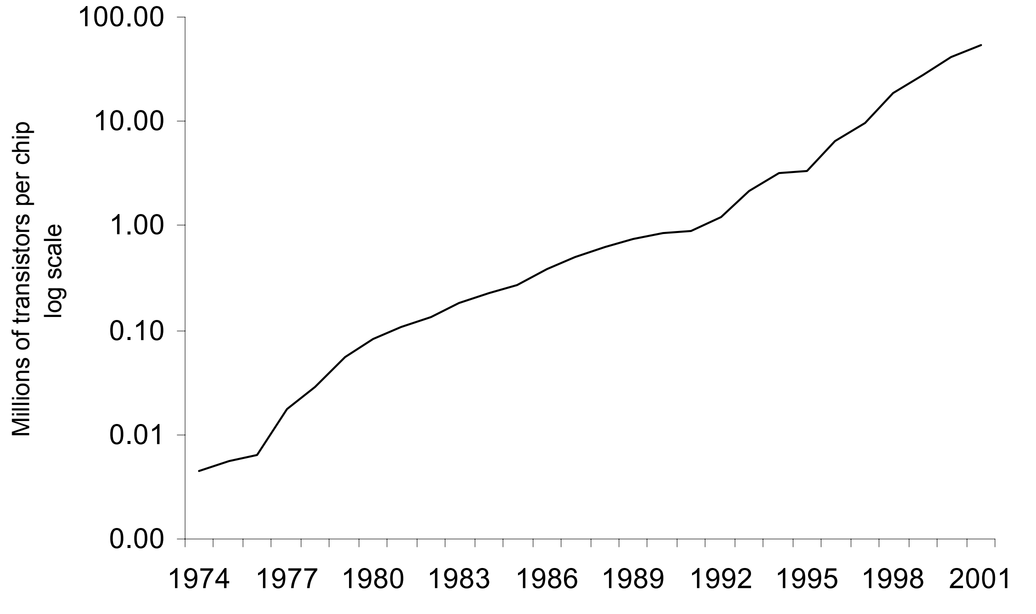
Table 3. Baseline Parameter Values

r	ε	c	q	g	λ
.07	1.3	75	486	.24	0.1

Table 4. Illustration of Change in Moore's Law

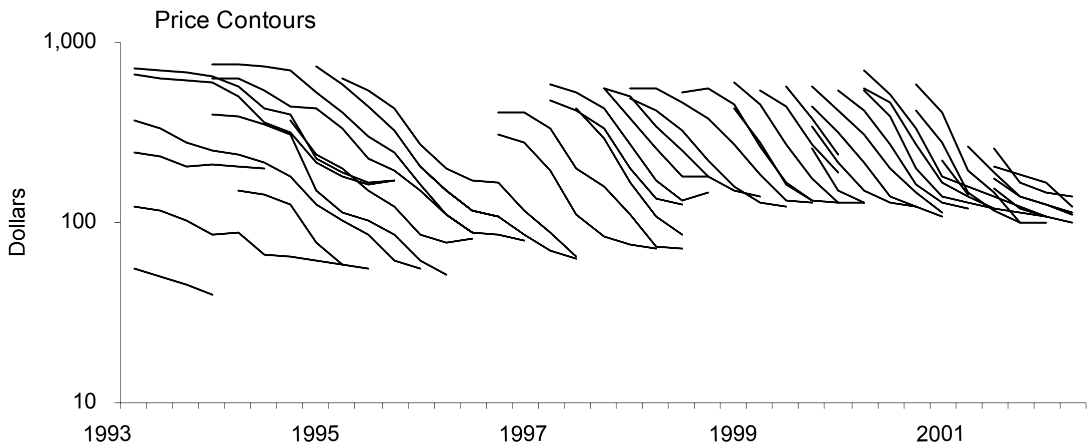
	Pentium I	Moore's Law		Contour Slopes	
	1993	Base Case	Increase	Base Case	Increase
Prices (dollars)					
Introduction	753	775	1070	772	839
Exit	119	325	325	210	210
Lifespan (years)	2.5	3.7	2.1	1.8	1.6
Growth rates (CAGR):					
No. of Transistors	24	24	58	74	90
Price Declines	-74	-24	-58	-74	-90

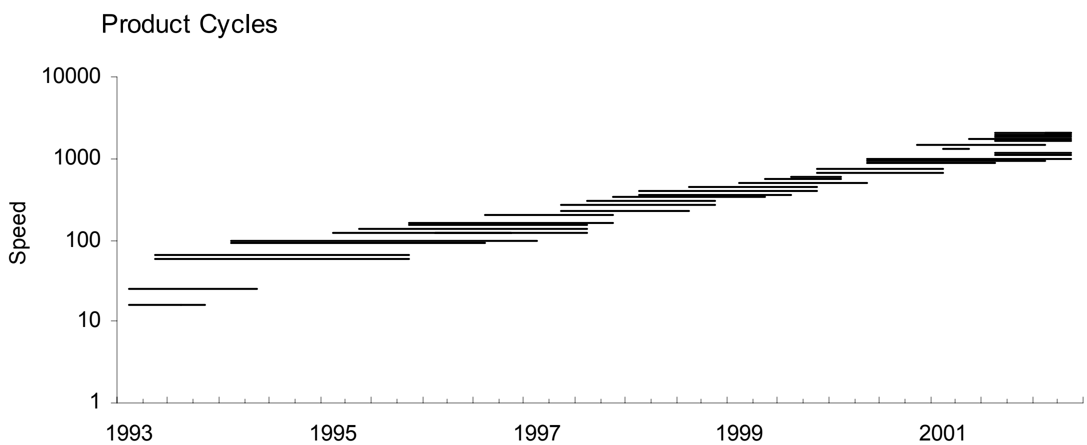
Figure 1. Moore's Law



Dots show dates of introduction and lines are interpolations between these dates.

Figure 2. Price Contours and Product Cycles for Intel's Desktop Microprocessor Chips, 1993-2002





Source: MicroDesign Resources, Inc.

Figure 3. Effect of an Increase in Moore's Law on Price Contours and Product Cycles