Research Spotlight

An Empirical Review of Methods for Temporal Distribution and Interpolation in the National Accounts

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THE Bureau of Economic Analysis has adopted a new method for the interpolation of quarterly and monthly estimates in the national accounts. The new method uses a variant of the Denton procedure.

National statistical agencies routinely face the task of compiling a large number of quarterly and monthly estimates using relatively complete annual data and less complete quarterly and monthly information from various indicators. Annual data are usually detailed and of high precision, providing the most reliable information on the overall level and long-term movement in the series. Quarterly or monthly data, while they are less detailed and of lower precision, provide timely and explicit information about the short-term movement in a series.

The objective of interpolation in the national economic accounts is to use annual data to derive quarterly or monthly estimates that preserve as much as possible the short-term movement in the indicator series while still summing to a benchmark set by the annual data.[1] Typically, the annual sums of the quarterly or monthly indicator values are not consistent with the annual values. This means the two series may display inconsistent movements over time.

Over the years, the Bureau of Economic Analysis (BEA) has used a variety of techniques for interpolation of the national accounts.[2] These techniques yielded varying degrees of success, and analysts at BEA identified some technical challenges in the estimation process:

- Final quarterly or monthly series do not always follow the short-term movement in the indicator series.
- Final estimated series may exhibit a sharp decrease at the end of the series, known as the "flattening out" problem.
- Some procedures require forward extrapolation of the annual values before interpolation.

There has been a strong interest among BEA staff in finding better methods for interpolation. A BEA study on which this article was based evaluated various methods for interpolation and recommended methods that are suitable for routine practice in the national accounts.[3] The evaluation was conducted using a variety of series from the national economic accounts that were representative of those used in producing the national accounts.

This study found that the following methods yielded favorable results:

- The modified Denton proportional first difference method for annual series with indicators (Denton 1971; Helfand et al. 1977)
- The first difference smoothing method by Boot, Feibes, and Lisman (1967) for annual series with no

1. In statistics, when quarterly or monthly estimates of flow or index variables are derived, the process is known as temporal distribution. When estimates of stock variables are derived, the process is known as interpolation. In this article the term "interpolation" generally refers to both temporal distribution and interpolation.

2. These methods include the Bassie adjustment method (Bassie, 1958), the minimum constrained variance internal moving average method, the linear and the Lagrange polynomial interpolation procedures.

3. This study is available on the BEA Web site at <www.bea.gov/papers/pdf/chen_temp_aggregation_wp.pdf>.

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indicators

- The two-step modified Denton proportional first difference method for series that also have contemporaneous constraints (Denton 1971; Helfand et al. 1977; Eurostat 1999)

In the remainder of the article, we briefly describe the methods that were evaluated, outline the criteria used for the evaluation, present the summary estimation results, and discuss the methods that will be used for the U.S. national accounts.

Methods Evaluated

A variety of mathematical and statistical methods have been developed for interpolation. The choice of a particular method depends on the basic information available for estimation and the operational constraints imposed by the statistical agencies.

The interpolation process in the national income and product accounts (NIPAs) occurs during the annual revision that is usually released in late July. According to the current revision policy, quarterly or monthly estimates of the 3 most recent years are revised during annual revisions.[4] This revision policy results in time series samples of, at most, 3 years for estimation. Thus, we restrict this report to the evaluation of the following six mathematical methods:[5]

- The modified Denton additive first difference method developed by Denton (1971) and Helfand and others (1977) (Denton additive)
- The modified Denton proportional first difference method by Denton (1971) and Helfand and others (1977) (Denton proportional)
- The growth-rate preservation method developed by Causey (1981) and Trager (1982) (growth preservation)
- The first difference smoothing method developed by Boot, Feibes, and Lisman (1967) (first difference smoothing)
- The second difference smoothing method developed by Boot, Feibes, and Lisman (1967) (second difference smoothing)
- The Lagrange polynomial interpolation procedure (Lagrange polynomial)

For more details on these methods, see Cholette and Dagum (2006) and Chen (2007).

The first three methods are based on the principal of short-term movement preservation.[6] Accordingly, the estimated quarterly or monthly series should preserve the movement in the quarterly or monthly indicator series, because the movement in the indicator series is the only information available. Although all interpolation methods modify the indicator values so that the annual sums or averages of the resulting quarterly or monthly estimates are equal to the corresponding annual benchmarks, each variant of the short-term movement preservation method is distinguished by its objective.

Denton additive

The objective of the Denton additive method is to keep the difference between the estimated quarterly or monthly series and the indicator series as constant as possible, subject to annual benchmarks. The resulting final estimates tend to be parallel to the indicator values.

Denton proportional

The objective of the Denton proportional method is to keep the ratio of the estimated quarterly or monthly series to the indicator series as constant as possible under the annual constraints. The final estimates tend to have the same period-to-period growth rates as the indicator series.

Growth rate preservation

The growth rate preservation method is intended to keep the sample period-to-period growth rates of the estimated quarterly or monthly series as close as possible to those of the indicator series. Consequently, this model becomes nonlinear in the final estimates and must be solved iteratively. The final estimates also tend to have the same growth rate as the indicator series.

First difference smoothing

The first difference smoothing method is used for interpolation only when annual data are available. The basic idea is to consider the trend of the unknown quarterly or monthly estimates as a smooth mathematical function of time. Thus, the objective of the first

4. Annual data for the 3 most recent years are revised during the annual revision. Indicator values for the 3 most recent years are revised depending on the availability of new data.

5. Although a variety of time series methods, such as ARIMA and regression-based methods, have been developed for interpolation, they generally require a time series sample that is much longer than 3 years to produce sensible results. For estimation results of time series methods using longer time series from the national accounts, see Chen (2007).

6. The original Denton methods had an erroneously specified initial condition, which introduces a transient movement at the beginning of the estimated series. The modified Denton methods use the correctly specified initial condition (Cholette and Dagum 2006).

difference smoothing method is to minimize the sample period-to-period change in the final quarterly or monthly estimates.

Second difference smoothing

The second difference smoothing method is also used when only annual data are available. The objective of this method is to keep the period-to-period change in the final quarterly or monthly estimates as linear as possible.

Lagrange polynomial

The Lagrange polynomial method is a purely mathematical procedure. It takes a collection of annual values and constructs a Lagrange polynomial function that passes through these annual values. This procedure has some known drawbacks. One drawback is that the interpolated values based on a certain assumed degree of polynomial function could sharply disagree with the actual values of the function because of a lack of information about the function globally. Moreover, under the 3-year annual revision policy, forward extrapolation of annual values is needed in some cases before interpolation can be conducted. The Lagrange polynomial method was evaluated because it has been frequently used for interpolation in the national accounts since the 1990s.[7]

Evaluation Criteria

In the national accounts, there are four main scenarios analysts face when interpolating data:

- Both annual and indicator data are available.
- Only annual data are available and all values are positive.
- Only annual data are available and some values are negative.
- Contemporaneous constraints are present; a contemporaneous constraint is defined as an accounting relationship through which a number of final quarterly or monthly series are linked. For example, quarterly taxes on production and imports are estimated for 16 industries; the contemporaneous constraint is that each quarter, they must sum to the quarterly total of taxes on production and imports.

Our objective was to find suitable methods capable of handling these four cases. In general, the most suitable methods for interpolation in the national accounts should generate final quarterly or monthly estimates that best preserve the period-to-period movement in the indicator series, if available, under the annual aggregation and, if present, contemporaneous constraints. We evaluate the six interpolation methods described previously according to certain statistical and operational criteria. Because indicator data may not be available, the evaluation criteria are specified accordingly.

If quarterly or monthly indicator data are available for interpolation, the evaluation criteria are as follows:

1. Annual aggregation constraints must be satisfied. For flow and index variables, the annual sums or averages of the quarterly or monthly estimates should be equal to the corresponding annual values; for stock variables, the estimate of the last quarter or month of each year should be equal to the corresponding annual value.

2. Final estimated series should preserve the period-to-period movement in the indicator series as much as possible.

3. Final quarterly or monthly estimates should exhibit minimum distortion to the period-to-period percentage change in the indicator series at the breaks between the years, that is, for quarterly series, from the fourth quarter to the following first quarter and, for monthly series, from November to the following February. There should be a minimum of distortion at the end of the sample as well. Some interpolation methods have been shown to have a tendency to exhibit some distortion of the period-to-period percentage change in the indicator series at these breaks (Hood 2005).

4. If final estimates are linked to previously benchmarked series, the newly estimated series should not exhibit an abrupt increase or decrease in the linking period.

5. Contemporaneous constraints, if present, must be satisfied.

If quarterly or monthly indicator data are not available for interpolation, evaluation should be based on the smoothness of the final quarterly or monthly estimates under the annual aggregation constraints. Criteria 1, 4, and 5 above remain unchanged, but criteria 2 and 3 are modified as follows:

2. Sample period-to-period change in final quarterly or monthly estimates should be minimized to assure smoothness.

3. Final estimates should not exhibit a sharp increase or decrease in the last period or at the breaks between the years in the sample.

7. The version of the Lagrange polynomial method in the software program that many BEA analysts use was not designed for interpolations of stock series. Hence, it was not tested on stock series.

Estimation Results

We selected 60 series, covering a period of 8 to 12 years, to represent the four scenarios previously discussed. Because the choice of a method depends on the basic information available for estimation, we separate the 60 series into two categories: (1) annual series with indicators, and (2) annual series with no indicators.

Of the 60 series, 45 series have indicators, 14 of which have only annual aggregation constraints, and the remaining 31 series have both annual aggregation and contemporaneous constraints. Of the 31 series that have contemporaneous constraints, 16 are taxes on production and imports, and 15 are transfer payments to the government. Quarterly total taxes on production and quarterly total transfer payments are, respectively, the contemporaneous constraint of each group. Of the 15 series that have no indicators, five have some negative annual values.[8] In this section, we present a comparative summary of the estimation results. More detailed results are in Chen (2007) and Chen and others (2007).

Results for annual series with indicators

To evaluate whether annual aggregation constraints are satisfied (criterion 1), we calculate the sample average proportional annual discrepancy of the interpolated values. The proportional annual discrepancy is defined as the annual sum of the interpolated values divided by the corresponding annual value. For index variables, we calculate the annual average of estimated quarterly or monthly values divided by the annual value.

To evaluate how well the movement in the indicators was tracked (criteria 2–4), we calculate the absolute difference in period-to-period percentage changes between final quarterly or monthly estimates and the indicator values.

To evaluate whether contemporaneous constraints were satisfied (criterion 5), we calculate the sample average proportional contemporaneous discrepancy with respect to final estimates. The proportional contemporaneous discrepancy is defined as the quarterly or monthly sum of the related series divided by the corresponding quarterly or monthly total.

We use six statistics to compare final estimates generated by each method according to the previously specified criteria:

- **Average proportional annual discrepancy.** It must be equal to one to satisfy the constraint.
- **Average absolute difference in period-to-period percent change.** The closer to zero the statistic is, the better the method preserves indicator movement.
- **Average absolute difference in period-to-period percent change at all breaks between the years.** The closer to zero the statistic is, the smaller the distortion to the indicator movement at the breaks between the years.
- **Average absolute difference in period-to-period percent change in the last period.** The closer to zero the statistic is, the better the method preserves indicator movement.
- **Average absolute difference in period-to-period percent change in the linking period.** The closer to zero the statistic is, the smoother the linking is.
- **Average proportional contemporaneous discrepancy.** It must be equal to one to satisfy the constraint.

The first two statistics are used to evaluate criteria 1 and 2. The next two are used to evaluate criteria 3; and the last two are used to evaluate criteria 4 and 5.

We compare the averages of these six statistics for all estimated series in each of the three groups: (1) 14 series with no contemporaneous constraints; (2) 16 series of taxes on production and imports; and (3) 15 series of transfer payments. The latter two groups have contemporaneous constraints.

The Denton additive, Denton proportional, growth preservation, and Lagrange polynomial methods were used to interpolate the 14 annual series that have indicators but no contemporaneous constraints. The results are shown in the top panel of table 1.

The Denton proportional, growth preservation and Lagrange polynomial methods were used to estimate the 31 series with contemporaneous constraints. The results are shown in the middle and lower panels of table 1. The Denton additive method was not used to estimate the 31 series with contemporaneous constraints, because of the occasional volatile patterns exhibited in the 14 estimated series that have no contemporaneous constraints.

The 31 series with contemporaneous constraints were estimated using a two-step method. In the first step, each series was estimated using each method. The estimates were then used as the preliminary estimates in the second step, in which the Denton proportional method was used to remove contemporaneous discrepancies. Linking is not tested for these series,

8. Negative values arise for some series because they are defined as the difference between two positive values.

because previously benchmarked data are not available. The proportional contemporaneous discrepancies were computed using estimates from both first and second step estimation. Table 1 shows the averages of the proportional contemporaneous discrepancies from the first and second step estimations.

The estimation results showed the following:

- Final estimates from the methods evaluated satisfied the annual aggregation constraints.
- The Denton proportional method outperformed the other methods in preserving the period-to-period movement in the indicator series.
- The Denton additive method occasionally generated estimates that exhibit large distortions to the movement in the indicator series.
- The Lagrange polynomial method had the weakest performance in preserving short-term movement in the indicator series.
- The two-step Denton proportional method outperformed the other methods for estimating series with both annual aggregation and contemporaneous constraints.

The strong overall performance of the Denton proportional method in preserving short-term movement is evident from the minimum values of all average absolute differences in period-to-period percent changes computed for the 14 series with no contemporaneous constraints and the 16 series of taxes on production and by being a close competitor to the growth rate preservation method for the 15 series of transfer payments.

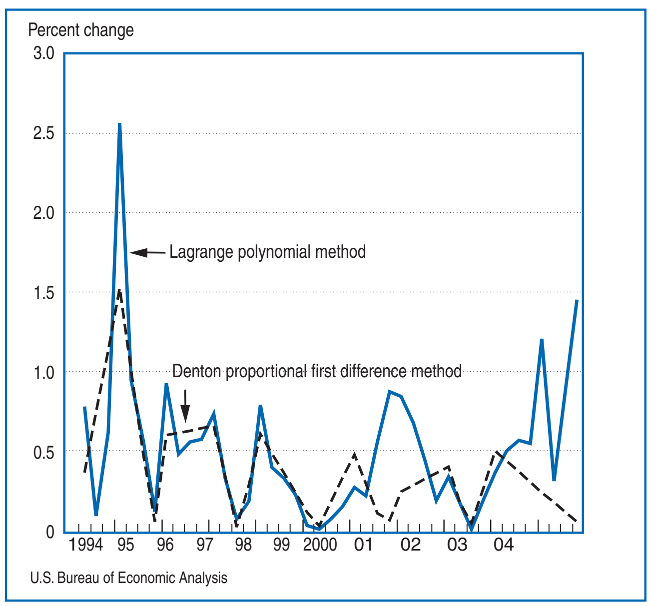
Table 1, which presents summary test statistics, confirms the strong overall performance of the Denton proportional method. The overall weak performance of the Lagrange polynomial procedure can be seen in chart 1, where the final Lagrange polynomial estimates generated larger distortions at the breaks between the years, and at the beginning and ending period of the sample. Moreover, table 1 shows that Lagrange polynomial estimates resulted in some level of contemporaneous discrepancy from the first step estimation.

Table 1. Comparison of Alternative Interpolation Methods for Annual Series With Indicators

Evaluation statistic	Interpolation methods			
	Lagrange polynomial	Growth preservation	Denton proportional	Denton additive
14 series without contemporaneous constraint				
1	1.0000	1.0000	1.0000	1.0000
2	0.0280	0.0140	**0.0135**	0.8742
3	0.0094	0.0168	**0.0159**	0.4731
4	0.0129	0.0030	**0.0025**	0.1464
5	*	0.0307	**0.0291**	0.0824
16 series of taxes with contemporaneous constraint				
1	1.0000	1.0000	1.0000	n.a.
2	0.0089	0.0089	**0.0088**	n.a.
3	0.0521	0.0105	**0.0104**	n.a.
4	0.0042	0.0092	**0.0031**	n.a.
6	0.1547	0.0042	**0.0019**	n.a.
7	1.0000	1.0000	1.0000	n.a.
15 series of transfer payments with contemporaneous constraint				
1	1.0000	1.0000	1.0000	n.a.
2	0.0191	**0.0187**	0.0189	n.a.
3	0.0955	0.0222	**0.0221**	n.a.
4	0.0082	**0.0035**	0.0049	n.a.
6	0.2990	0.2106	**0.0018**	n.a.
7	1.0000	1.0000	1.0000	n.a.

* Statistic is not computed because of lack of previously benchmarked data.
n.a. Statistics are not computed, because Denton additive is not included in evaluation.
1. Proportional annual discrepancy, average ratio of the annual sum of the estimates to the annual values.
2. Difference in the average of the absolute period-to-period percentage change between the estimated and indicator series.
3. Difference in average absolute period-to-period percentage change at breaks between years.
4. Difference in absolute period-to-period percentage change in the last sample period.
5. Difference in absolute period-to-period percentage change for the first sample period linked to the previously benchmarked series.
6. Average contemporaneous discrepancy in stage 1 estimation.
7. Average contemporaneous discrepancy in stage 2 estimation.
NOTE. Numbers in bold are the smallest value of each statistic and indicate the method that best preserves the movement in the indicator series.

Chart 1. Absolute Difference in Quarterly Percent Change Between Estimated and Indicator Series, Crop Output, 1994–2004



Results for annual series with no indicator

The first and second difference smoothing methods and the Lagrange polynomial method are used to interpolate the annual series that have no quarterly or monthly indicators. For the 10 annual series that have no negative values, we evaluate the smoothness of the final quarterly or monthly estimates using the absolute period-to-period percent change of final estimated series. There are no contemporaneous constraints for these series, and linking is not evaluated, because previously benchmarked series are not available. Thus, four evaluation statistics are used to compare the final estimates generated by each method. The comparison is based on the following 4 mean statistics of the 10 estimated series:

- Average proportional annual discrepancy
- Absolute period-to-period percent change
- Absolute period-to-period percent change in the linking period
- Absolute period-to-period percent change in the last period

For the five series that have some negative annual values, the evaluation of smoothness is based on the absolute period-to-period level change, the absolute change in the linking period, and the absolute change in the last period.

The comparative summary results of the 10 series with no negative values are shown in the top panel of table 2, and the results for the five series with negative annual values are shown in the bottom panel of table 2.

Table 2. Mean Statistics for Evaluation of Alternative Methods for Interpolation of Annual Series with No Indicators

Evaluation statistic	Interpolation methods		
	Lagrange polynomial	First difference smoothing	Second difference smoothing
10 annual series with no negative values			
Proportional annual discrepancy	1.0000	1.0000	1.0000
Growth rate, period *t*	6.2834	**5.5831**	5.6455
Growth rate, linking period	12.5887	**9.1193**	9.7500
Growth rate, last period	3.1748	**0.9446**	4.2095
5 annual series with some negative values			
Proportional annual discrepancy	1.0000	1.0000	1.0000
Change in period *t*	24.7718	**24.6863**	26.2508
Change in linking period	8.4000	**5.1840**	19.4340
Change in last period	7.2000	**3.3500**	11.5825

NOTES. The proportional annual discrepancy refers to the average ratio of the annual sum of estimates to the annual values.
Growth rates are period-to-period growth rates.
Change refers to the change in period-to-period levels.
Numbers in bold are the smallest value of each statistic and indicate the method that produces the smoothest estimates.

The results show the following for series with no indicators, regardless of whether annual data have negative values:

- The first difference smoothing method produced the smoothest quarterly or monthly estimates, outperforming the Lagrange polynomial and the second difference smoothing methods.
- The second difference smoothing method tended to generate much larger estimates in the ending period and produce larger period-to-period percentage (or level) change in the beginning and ending periods.
- Lagrange polynomial estimates may exhibit a pattern of abrupt period-to-period percentage (or level) change throughout the sample.

Selected Methods for the National Accounts

Based on our results, BEA will use the following methods for routine interpolation of various series in the national income and product accounts:

- The Denton proportional method when both annual and indicator data are available[9]
- The first difference smoothing method when only annual data are available
- The two-step Denton proportional method when both temporal and contemporaneous constraints are present

The Denton proportional method was selected because it preserves the short-term fluctuations in the indicator series better when the fluctuations are distributed multiplicatively around the trend of the series, a frequently observed characteristic in the macroeconomic data.

In contrast, the Denton additive method results in a smooth additive distribution of the errors. Thus, this method tends to smooth away some of the period-to-period percentage changes in the indicator series. Consequently, the Denton additive method can seriously distort the short-term movements for series that show strong short-term variations.

The growth rate preservation method performed similarly to the Denton proportional method in our experiment. However, in general, the growth preservation method tends to adjust small rates of changes relatively more than large rates of changes in the indicator series, which is not a desirable property.

The Lagrange polynomial estimates may fail to

9. BEA is using Census Bureau software to interpolate using the modified Denton proportional first difference method.

preserve the period-to-period percentage change in the ending period and at the breaks between the years.

The two-step Denton proportional method has the advantage of removing contemporaneous discrepancies and thus eliminating the need for distributing contemporaneous discrepancies manually.

In sum, the selected methods collectively have the following advantages, which should strongly improve interpolated estimates:

- If indicator data are available, short-term movement in the indicator series is best preserved.
- The "flattening-out" problem at the end of the sample has been eliminated.
- Forward extrapolation of the annual values is not necessary.
- All types of variables—flow, stock, and index—can be interpolated.
- Newly revised series can be smoothly linked to previously benchmarked series.

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