Bureau of Economic Analysis
U.S. DEPARTMENT OF COMMERCE

Technical Document: Methodology for Preparing Single-Country Trade in Value Added (TiVA) Statistics

Introduction to TiVA Statistics

International trade statistics under the current international guideline of the System of National Accounts (SNA) record gross flows of goods and services each time they cross a border. Over the last several decades, the global economy has experienced tremendous globalization. Production and international trade of goods and services have been increasingly linked to global value chains (GVCs), which has significantly changed the nature and structure of international trade, especially with growing intermediate goods trade.[1] Globalization has led to increasingly complicated supply chains as intermediate goods (raw materials, for example) move strategically throughout the world as they are transformed step by step into final goods sold to end users. Existing data on gross trade flows continue to answer many important questions about the global economy; however, the growth of GVCs means that additional data are necessary for a more complete and more nuanced understanding of global trade.

Trade in Value Added (TiVA) statistics help complete the picture of global trade by tracing the value-added contributions of domestic industries and the role of imported content in the global supply chains that support domestic production. These data are prepared using input-output analytical techniques to estimate the sources of value in the domestic production of goods and services. This paper describes the methodology and prototype results of work being conducted by the Bureau of Economic Analysis in conjunction with the National Center for Science and Engineering Statistics (NCSES) of the National Science Foundation to prepare single country TiVA indicators derived from the U.S. supply-use tables for 2007 to 2020.

1. https://www.oecd-ilibrary.org/docserver/5k3v1trgnbr4-en.pdf?expires=1638973074&id=id&accname=guest&checksum=00CF2BDE148F9432CB10CE1AD0347E75

The paper is organized as follows: an introduction to TiVA, a methodology section describing steps for creating TiVA statistics using available expanded make and use tables and import matrices with five U.S. trade partners used for the TiVA publication, a summary of this publication, and a discussion of future studies and publications for BEA's single-country TiVA statistics.

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Overall Strategy

TiVA statistics are generally prepared in a multi-country supply-use framework. These multi-country tables are built using a collection of national-level supply-use tables as well as other data prepared by individual National Statistical Agencies that facilitate the integration of these tables into a single and consistent global supply-use table. Multi-country tables generally cannot be compiled by a single national statistical office from regular national accounts data due to the absence of necessary data about other countries. As a result, multi-country tables are typically compiled by international organizations or research institutes, which collect and integrate supply-use tables from the various countries.

The multi-country supply-use framework has the advantage that it allows a complete view from start to finish of a global value chain to be traced across multiple economies. Among other things, this allows the identification of value that is "double counted" in standard trade statistics as it crosses the same border multiple times as well as value that is exported and later returned to an economy embedded in imported products. However, there are also notable challenges to assembling multi-country supply-use tables, including inconsistent industry and product classification systems adopted by different countries, varying aggregation levels, and asymmetries in trade data across countries, as well as distributing import and export values among intermediate purchase and final uses for each destination country.

While the TiVA statistics described in this paper are based on single-country supply-use tables and don't trace out the full global value chain, they still provide many important insights and have several advantages over the multi-country framework. Most notably, single-country TiVA statistics for the United States can be prepared on a more timely basis and at a more detailed level of aggregation than multi-country tables, results are more consistent with official national accounts data for the United States, and tables can be customized to emphasize specific industries/products of particular interest for the U.S. economy. In addition, future research will examine the possibility of a hybrid model that incorporates some data from multi-country supply-use tables into the single-country framework in order to deliver some of the benefits from both approaches.

Expansion of Industry Detail

BEA's featured supply-use tables are published annually for 71 industries, or roughly a 3-digit North American Industry Classification System (NAICS) level of detail. The TiVA statistics presented here are generated using make, use, and import matrices prepared at an expanded 81-industry level of detail. This expansion provides increased detail, particularly in technology and aerospace manufacturing, medical supplies manufacturing, pharmaceuticals, information services, and research and development (table 1). In addition to the TiVA statistics themselves, BEA is also making the underlying 81-industry make, use, and import matrices available, which allows users to calculate other types of TiVA statistics not explicitly prepared as part of this release as well as to make use of these more detailed tables in other types of analysis. Looking forward, BEA will explore the possibility of publishing TiVA statistics and underlying supply-use tables at the 138-industry level currently available as underlying detail in BEA's GDP by industry suite of products.

Table 1. BEA Summary Industry Detail to TiVA Industry Detail

71-Industry	Description	81-Industry	Description
111CA	Farms	111CA	Farms
113FF	Forestry, fishing, and related activities	113FF	Forestry, fishing, and related activities
211	Oil and gas extraction	211	Oil and gas extraction
212	Mining, except oil and gas	212	Mining, except oil and gas
213	Support activities for mining	213	Support activities for mining
22	Utilities	22	Utilities
23	Construction	23	Construction
321	Wood products	321	Wood products
327	Nonmetallic mineral products	327	Nonmetallic mineral products
331	Primary metals	331	Primary metals
332	Fabricated metal products	332	Fabricated metal products
333	Machinery	333	Machinery
334	Computer and electronic products	3342	Communications equipment manufacturing
334	Computer and electronic products	3344	Semiconductor and other electronic component manufacturing
334	Computer and electronic products	3345	Navigational, measuring, electromedical, and control instruments manufacturing
334	Computer and electronic products	334AO	Other computer and electronic product manufacturing
335	Electrical equipment, appliances, and components	335	Electrical equipment, appliances, and components
3361MV	Motor vehicles, bodies and trailers, and parts	3361MV	Motor vehicles, bodies and trailers, and parts
3364OT	Other transportation equipment	3364	Aerospace product and parts manufacturing
3364OT	Other transportation equipment	3365AO	All other transportation equipment manufacturing
337	Furniture and related products	337	Furniture and related products
339	Miscellaneous manufacturing	3391	Medical equipment and supplies manufacturing
339	Miscellaneous manufacturing	3399	Other miscellaneous manufacturing
311FT	Food and beverage and tobacco products	311FT	Food and beverage and tobacco products
313TT	Textile mills and textile product mills	313TT	Textile mills and textile product mills
315AL	Apparel and leather and allied products	315AL	Apparel and leather and allied products
322	Paper products	322	Paper products
323	Printing and related support activities	323	Printing and related support activities
324	Petroleum and coal products	324	Petroleum and coal products
325	Chemical products	3254	Pharmaceutical and medicine manufacturing
325	Chemical products	325AO	Other chemical products
326	Plastics and rubber products	326	Plastics and rubber products
42	Wholesale trade	42	Wholesale trade
441	Motor vehicle and parts dealers	441	Motor vehicle and parts dealers
445	Food and beverage stores	445	Food and beverage stores
452	General merchandise stores	452	General merchandise stores
4A0	Other retail	4A0	Other retail
481	Air transportation	481	Air transportation
482	Rail transportation	482	Rail transportation
483	Water transportation	483	Water transportation
484	Truck transportation	484	Truck transportation
485	Transit and ground passenger transportation	485	Transit and ground passenger transportation
486	Pipeline transportation	486	Pipeline transportation
487OS	Other transportation and support activities	487OS	Other transportation and support activities
493	Warehousing and storage	493	Warehousing and storage
511	Publishing industries, except internet (includes software)	5111	Newspaper, periodical, book, and directory publishers
511	Publishing industries, except internet (includes software)	5112	Software publishers
512	Motion picture and sound recording industries	512	Motion picture and sound recording industries
513	Broadcasting and telecommunications	513	Broadcasting and telecommunications
514	Data processing, internet publishing, and other information services	518	Data processing, hosting, and related services
514	Data processing, internet publishing, and other information services	519	Other information services
521CI	Federal Reserve banks, credit intermediation, and related activities	521CI	Federal Reserve banks, credit intermediation, and related activities
523	Securities, commodity contracts, and investments	523	Securities, commodity contracts, and investments
524	Insurance carriers and related activities	524	Insurance carriers and related activities
525	Funds, trusts, and other financial vehicles	525	Funds, trusts, and other financial vehicles
HS	Housing	HS	Housing
ORE	Other real estate	ORE	Other real estate
532RL	Rental and leasing services and lessors of intangible assets	532RL	Rental and leasing services and lessors of intangible assets
5411	Legal services	5411	Legal services
5415	Computer systems design and related services	5415	Computer systems design and related services
5412OP	Miscellaneous professional, scientific, and technical services	5417	Scientific research and development services
5412OP	Miscellaneous professional, scientific, and technical services	541AO	All other professional, scientific, and technical services
55	Management of companies and enterprises	55	Management of companies and enterprises
561	Administrative and support services	561	Administrative and support services
562	Waste management and remediation services	562	Waste management and remediation services
61	Educational services	61	Educational services
621	Ambulatory health care services	621	Ambulatory health care services
622	Hospitals	622	Hospitals
623	Nursing and residential care facilities	623	Nursing and residential care facilities
624	Social assistance	624	Social assistance
711AS	Performing arts, spectator sports, museums, and related activities	711AS	Performing arts, spectator sports, museums, and related activities
713	Amusements, gambling, and recreation industries	713	Amusements, gambling, and recreation industries
721	Accommodation	721	Accommodation
722	Food services and drinking places	722	Food services and drinking places
81	Other services, except government	81	Other services, except government
GFGD	Federal general government (defense)	GFGD	Federal general government (defense)
GFGN	Federal general government (nondefense)	GFGN	Federal general government (nondefense)
GFE	Federal government enterprises	GFE	Federal government enterprises
GSLG	State and local general government	GSLGE	State and local government educational services
GSLG	State and local general government	GSLGAO	All other state and local general government
GSLE	State and local government enterprises	GSLE	State and local government enterprises

Regional Detail

In addition to expanded industry detail, the supply-use framework has been extended for purposes of TiVA calculations to include regional breakouts of the import and export vectors. More specifically, bilateral trade data from BEA's International Transactions Accounts have been used to separately identify trade with Canada, China, Europe, Mexico, and the rest of the world (ROW)[2]. Looking forward, BEA will examine the possibility of expanding these statistics to include additional countries and regions.

Upstream and Downstream Perspectives

The newly published TiVA estimates can be broadly classified as taking either an upstream or downstream perspective on the analysis of U.S. gross exports. From an upstream perspective, the TiVA estimates examine gross exports from the starting point of the exports themselves. Through the application of the single-country framework, the gross exports can be decomposed into domestic value added or imported content that is embedded in gross exports. This is referred to as upstream because the analysis is from the "finished product" or gross exports and is looking up the supply chain to analyze which industries contributed value added and which regions contributed imported content to the production of those exports. From a downstream perspective, the TiVA estimates examine an industry's production of value added and trace that value added as it flows down the supply chain to determine which industries rely on that value added in their production of output, and therefore, which industry's exports the value added is embedded in. This is referred to as downstream as the analysis is from the start of the supply chain and is looking down it to analyze where the value added flows to.

Using the production of a car as an example, an upstream perspective of TiVA would start with the gross exports of the car manufacturing industry. Some of the value added embedded in those exports can be attributed to the car manufacturing industry itself, but there are other inputs needed to manufacture a car. Ore is mined and then refined into metals that are used in the production of the car. Silicon is manufactured into computer chips which are inserted into the car. Each of these steps of production will have both domestically produced inputs and imported content that all flow into the gross exports of cars. From an upstream perspective, TiVA estimates analyze every step of the supply chain to account for the value added that is produced in each required input and therefore is embedded in the final export of the car.

From a downstream perspective, the starting point is the value added produced by an industry. For example, take the mining industry's value added. As discussed above, some of that value added will flow to the car manufacturing industry through the refining and manufacture of metal products. Other value added will flow

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2. The TiVA services trade statistics consist primarily of statistics published in BEA's U.S. Trade in Services statistics, Table 2.3. U.S. Trade in Services, by Country or Affiliation and by Type of Service. Because some cells in BEA's trade in services table are suppressed, the TiVA statistics include imputed values for the suppressed cells. The methodology used to impute suppressed cell values relied on information from surrounding cells in the published Table 2.3. No information on the underlying suppressed values was used for imputation.

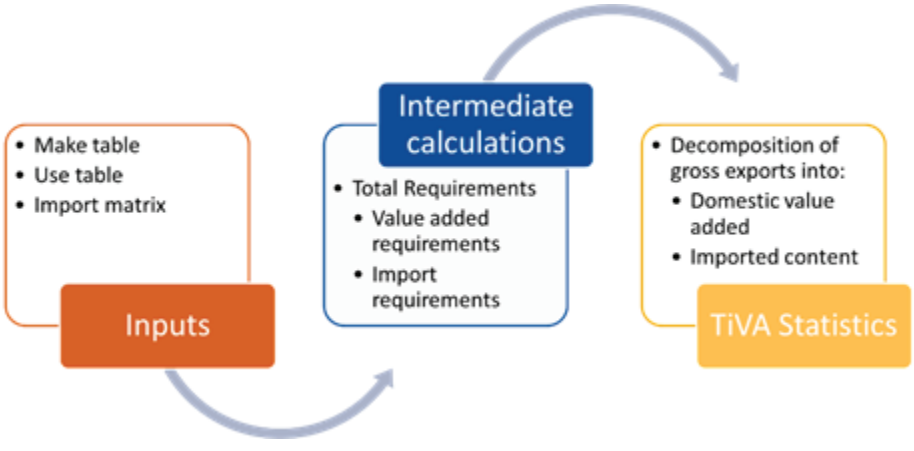
   The TiVA goods trade statistics consist of statistics publicly available through the U.S. Census Bureau's online database, USA Trade Online.

into the petroleum and coal product manufacturing industry and then on into utilities as those products are used to create electric power generation. A downstream perspective tracks an industry's value added as it flows throughout the supply chain of U.S. production to show how an industry's production of value added contributed to U.S. gross exports.

With these terms in mind, TiVA estimates can be used to answer a variety of questions based on the perspective you take. From a downstream perspective, TiVA estimates can answer questions such as "how important is the mining industry's production in the gross exports of the car manufacturing industry?" or "how much of this industry's value added is purchased abroad in U.S. gross exports?" From an upstream perspective, TiVA estimates can answer questions such as "what industries are most important in the production of car manufacturing's gross exports" or "how dependent is this industry on foreign supply chains in the production of their gross exports?"

Methodology

BEA's prototype TiVA statistics are produced using make, use, and import matrices after redefinitions from BEA's Annual Industry Accounts and data on bilateral trade in goods and services from BEA's International Transactions Accounts. Using these data as a starting point, the estimation process takes place in three main steps. First, certain conceptual modifications are made to the initial inputs to extend their usefulness in the context of TiVA. Second, the adjusted input tables are used to create the total domestic requirements, value added requirements, and import requirements matrices from which the TiVA statistics are built.[3, 4] Third, the requirements tables are used to create the TiVA statistics themselves. A complete derivation of the TiVA statistics is located in the appendix.



3. For information on input-output statistics, please see "A Primer on BEA's Industry Accounts": https://www.bea.gov/sites/default/files/methodologies/industry_primer.pdf#page=3

4. For further information on input-output statistics, please see "Concepts and Methods of the U.S. Input-Output Accounts": https://www.bea.gov/sites/default/files/methodologies/IOmanual_092906.pdf

Conceptual Adjustments

Before developing trade-in-value added statistics, the underlying make, use, and import matrices were adjusted in several ways to extend their usefulness for analyzing global value chains. First, the level of detail in the make, use, and import matrices was expanded from 71 industries to 81 industries as described previously.

Next, the import matrix was broken out into separate matrices for Canada, China, Europe, Mexico, and the rest of the world (ROW). This was accomplished using published data on bilateral trade in goods and services from BEA's international transactions accounts. As some cells in these published tables are suppressed, imputations were made by applying ratios from surrounding years where unsuppressed data were available.

Finally, the make and use tables were adjusted to reallocate most value from the "other" row to the commodities outlined in table 2. Unlike the rest of the table, the "other" row is not primarily comprised of related goods or services produced using similar input structures; rather, it is an adjustment row designed to account for things like direct domestic purchases abroad and direct foreign purchases within the United States. The new treatment reallocates value in this adjustment row to specific commodities whose input structures can be traced through the supply-use framework. For example, the "other" row includes an adjustment to properly account for expenditures by foreigners travelling in the United States, and this adjustment value has been reallocated to the accommodation; food services and drinking places; air transportation; and performing arts, spectator sports, museums, and related activities commodities. These changes improve TiVA estimates by accounting for the supply chain linkages necessary to provide services to foreign travelers in the United States. BEA continues to research the best approach for reallocating the "other" row and expects to incorporate improvements to this process in future versions of the TiVA statistics and eventually to bring these changes into BEA's standard supply-use tables.

Table 2. Reallocation of Values from Commodity "Other"

Value Reallocated to Commodities	
Code	**Description**
23	Construction
311FT	Food and beverage and tobacco products
315AL	Apparel and leather and allied products
325	Pharmaceutical and medicine manufacturing
334	Other computer and electronic product manufacturing
339	Other miscellaneous manufacturing
481	Air transportation
487OS	Other transportation and support activities
711AS	Performing arts, spectator sports, museums, and related activities
721	Accommodation
722	Food services and drinking places
GFGN	Federal general government (nondefense)
HS	Housing

Requirements Matrices

The next step in preparing the TiVA statistics is to calculate requirements matrices based on the adjusted make, use, and import matrices.[5] The first step in this process is to prepare a domestic use table. The domestic use table is the use table less the imports matrix. Next, a commodity-by-industry domestic direct requirements table is calculated from the domestic use table by dividing industry use of each commodity by the industry's gross output. This table shows the direct domestic production of each commodity (row) used or required for each dollar of gross output produced by an industry (column).

Next, an industry-by-commodity market share matrix is calculated from the make table by dividing production of each commodity by an industry (row) by total output of each good or service (column). The resulting table shows the share of output for a given commodity produced by each industry. Then, a symmetric industry-by-industry domestic direct requirements table is calculated by multiplying the industry-by-commodity market share matrix by the commodity-by-industry domestic direct requirements table. Finally, the industry-by-industry domestic total requirements table is calculated as the inverse of an industry-by-industry identity matrix less the industry-by-industry domestic direct requirements table. Each cell in this table shows total domestic production required from the industry in the row at all stages of the supply chain per dollar of output delivered to final use by the industry in the column.

Multiplying the industry-by-industry domestic total requirements table by a diagonal matrix of value added to gross output ratios yields the value added requirements table. Each cell in the value added requirements table shows how much value added from the industry in the row is embedded in one dollar of gross output of the industry in the column.

Similarly, multiplying the industry-by-industry domestic total requirements table by a diagonal matrix of import to gross output ratios yields the import requirements table. Each cell in this table shows the quantity of goods or services imported by the industry in the row embedded in one dollar of gross output of the industry in the column.

TiVA Statistics

Domestic value added in U.S. exports is calculated by multiplying the value-added requirements table by a diagonal matrix of industry exports. Industry exports are created by multiplying exports by commodity by the market share matrix. This procedure is repeated using industry exports to each trading partner to generate statistics for each purchasing region. Similarly, foreign content in U.S. exports is calculated by multiplying the import requirements table by a diagonal matrix of industry exports. This procedure is also repeated using an import requirements table specific for each trading partner. This step yields estimates of foreign content in U.S. exports by trading partner. Different TiVA statistics can be calculated by summing results in different ways from the detailed matrices obtained from the multiplication processes described above.

5. https://www.bea.gov/help/faq/32

TiVA statistics can provide a new perspective on traditional gross measures of exports and imports. BEA's single-country TiVA statistics can break out not only the foreign and domestic components of trade data, but also allow for a better look at the industry contributions to the exports of goods and services. For example, in gross trade statistics, the export of a car manufactured in the U.S. would be attributed entirely to the manufacturing sector. With a TiVA perspective, we can further decompose the exports of car manufacturing to show the various industries that contributed value added to that export. Continuing the car example, the input structure for motor vehicle manufacturing is not entirely composed of manufacturing inputs. There are also inputs from non-manufacturing industries such as legal services, research and development, power generation, and much more that are all necessary for the manufacture of a car. Furthermore, some of the inputs required to produce a car are purchased from foreign sources, and do not represent U.S. production. BEA's single-country TiVA framework allows us to decompose the various contributing industries and countries embedded within U.S. exports and provides a wealth of detail on U.S. imports and exports with trading partners.

Summary

TiVA estimates offer an interesting new way to understand how global supply chains and traditional trade estimates are related. When used in tandem with traditional trade statistics, they offer a wealth of detail and insights into the underpinnings of international trade estimates. BEA's new single-country TiVA statistics help show the importance that service-producing industries have on the production of goods and services within the United States and offer new avenues for analyzing the production of output for gross exports or final demand. The ability to distinguish between domestic and imported content in U.S. gross exports allows for analysis into the upstream global integration of industries, and estimates of how much of an industry's value added is ultimately exported show a downstream perspective on how globally integrated an industry is. The new estimates can be seen as a complimentary set of statistics to traditional gross trade estimates and provide an avenue for more detailed analysis of the value embedded in gross trade estimates.

Upcoming Research and Publications

These new statistics represent completion of the first major milestone at the end of the first year in a 3-year collaboration with the National Science Foundation. An in-depth review of the data providing an analysis of the TiVA statistics and trends will be published in a later white paper.

In 2022, BEA will explore increasing industry detail from 81 to 140 industries with a continued focus on R&D intensive industries and information communication technology. This will further enhance analysis of TiVA statistics by allowing a more granular view of industrial linkages.

In 2023, BEA will begin exploring expansion of the statistics to the level at which BEA publishes its quinquennial benchmark tables—approximately 400 industries. In addition to expanded industry detail, BEA will also explore increasing the number of regions presented in the results.

BEA is also researching the preparation of extended supply-use tables to account for firm heterogeneity within U.S. industries. This effort is focused on disaggregating industries within the supply-use framework into domestically controlled multinational enterprises (MNEs), foreign-controlled MNEs, and non-MNEs. Once completed, the TiVA framework will be expanded to incorporate these new data as well.

User comments on the methodology and data presentation, suggestions on where BEA should focus future industry and regional expansions, and any other comments are encouraged and welcome. Please send feedback to GVC@bea.gov.

Appendix: Mathematical Derivation of NSF TiVA

From make and use tables, the following are defined:

^: A symbol that, when placed over a vector, indicates a square matrix in which the elements of the vector appear on the main diagonal and zeros elsewhere.

t: Transposition operator.

q: A column vector in which each entry shows the total amount of each commodity's output.

g: A column vector in which each entry shows the total amount of each industry's output.

U: Intermediate portion of the use matrix in which the column shows for a given industry the amount of each commodity it uses, including noncomparable imports and used and secondhand goods. This is a commodity-by-industry matrix.

V: Make matrix in which the column shows for a given commodity the amount produced in each industry. This is an industry-by-commodity matrix. V has columns showing only zero entries for noncomparable imports and used and secondhand goods.

M: Intermediate portion of the import matrix in which the column shows for a given industry the amount of imported commodity it uses. This is a commodity-by-industry matrix.

U_d: Domestic intermediate inputs in which the column shows for a given industry the domestically produced amount of each commodity it uses, including noncomparable imports and used and secondhand goods. This is a commodity-by-industry matrix.

$$\boldsymbol{U_d = U - M} \quad (1)$$

B_d: Direct domestic input coefficients matrix in which entries in each column show the domestically produced amount of a commodity used by an industry per dollar of output of that industry. This is a commodity-by-industry matrix.

$$\boldsymbol{B_d = U_d \hat{g}^{-1}} \quad (2)$$

D: A matrix in which entries in each column show, for a given commodity, the proportion of the total output of that commodity produced in each industry. In the model, it is assumed that each commodity is produced by the various industries in fixed proportions. This is an industry-by-commodity matrix. D is also referred to as the market share matrix or transformation matrix.

$$\boldsymbol{D = V\hat{q}^{-1}} \quad (3)$$

i: Unit (summation) vector column containing only l's.

I: Identity matrix, where $\mathbf{I} = \hat{\boldsymbol{i}}$.

f: A column vector in which each entry shows the total final demand purchases for each commodity from the use table.

f_i: A column vector in which each entry shows the imported final demand purchases for each commodity from the import matrix.

f_d: A column vector in which each entry shows the domestically produced final demand purchases for each commodity from the import matrix.

$$\boldsymbol{f_d} = \boldsymbol{f} - \boldsymbol{f_i} \qquad (4)$$

e: A column vector in which each entry shows an industry's value added.

m: A column vector in which each entry shows imports purchased by an industry as intermediate inputs.

$$\boldsymbol{m} = (\boldsymbol{i}^t \boldsymbol{M})^t \qquad (5)$$

From the above definitions, the following identities are derived:

$$\boldsymbol{q} = \boldsymbol{U_d}\,\boldsymbol{i} + \boldsymbol{f_d} \qquad (6)$$

$$\boldsymbol{g} = \boldsymbol{V}\boldsymbol{i} \qquad (7)$$

Identity (6) shows that the row sum of a domestic use table equals commodity output.
Identity (7) shows that the row sum of the make table equals industry output.

From (2) and (6), we derive:

$$\boldsymbol{q} = \boldsymbol{B_d}\,\boldsymbol{g} + \boldsymbol{f_d} \qquad (8)$$

From (3) and (7), we derive:

$$\boldsymbol{g} = \boldsymbol{D}\boldsymbol{q} \qquad (9)$$

Substituting (9) into (8) and solving for *q* gives:

$$q = B_d(Dq) + f_d$$

$$(I - B_d D)q = f_d$$

$$\boldsymbol{q = (I - B_d D)^{-1} f_d} \quad (10)$$

The matrix $\boldsymbol{(I - B_d D)^{-1}}$ from equation (10) is known as the commodity-by-commodity domestic requirements matrix and it shows the output on a per-dollar basis the various commodity output that on average the economy produces in order to provide commodities to final users. Note that equation (10) also includes the identity that the commodity-by-commodity domestic requirements multiplied by domestic final demand equals commodity output.

Substituting (8) into (9) and solving for *g* gives:

$$g = D(B_d g + f_d)$$

$$(I - DB_d)g = Df_d$$

$$\boldsymbol{g = (I - DB_d)^{-1} Df_d} \quad (11)$$

The matrix $\boldsymbol{(I - DB_d)^{-1}}$ from equation (11) is known as the industry-by-industry domestic requirements matrix and it shows the output on a per-dollar basis that each industry on average produces in order to provide an industry's commodities to final users. The vector Df_d is a final demand vector where each entry shows the final demand for a domestic industry's output. Note that equation (11) also includes the identity that the industry-by-industry domestic requirements multiplied by final demand from domestic industries equals industry output.

Pre-multiplying both sides of equation (10) by D and substituting from equation (9) yields:

$$\boldsymbol{g = D(I - B_d D)^{-1} f_d} \quad (12)$$

The matrix $\boldsymbol{D(I - B_d D)^{-1}}$ from equation (12) is known as the industry-by-commodity domestic requirements matrix and it shows, on a per-dollar basis that each industry on average produces in order to provide products to final users. Note that equation (12) also includes the identity that the industry-by-commodity domestic requirements multiplied by domestic final demand equals industry output.

Up to now these calculations mirror the domestic requirements calculations from BEA's published tables. As a reminder these calculations use hybrid technology approach that embeds both the industry-technology assumption (ITA) and the commodity-technology assumption (CTA) methods.[6] The mathematical derivation follows the ITA mathematical derivation, but the source tables used are after redefinition tables that are calculated based on the CTA. As a result of utilizing this hybrid approach, these requirements tables are a more effective tool for analyzing supply-chain relations among industries.

The next steps involve generating two sets of tables that are not published by BEA: value added requirements and bilateral import requirements. It is from value added and bilateral import requirements that most Trade in Value Added (TiVA) statistics are computed. Note that value added requirements can be most naturally computed from either the industry-by-industry domestic requirements or the industry-by-commodity domestic requirements. What follows is a computation based on the industry-by-commodity domestic requirements matrix.

To compute value added requirements, we must first create a diagonalized matrix of value added to output ratios by industry. This can also be formulated as a diagonal matrix of industry value added multiplied by the inverse of a diagonal matrix of industry output.

$$\hat{e}\,\hat{g}^{-1}$$

Pre-multiplying the industry-by-commodity domestic requirements matrix $\boldsymbol{D(I - B_d D)^{-1}}$ by the above matrix yields the value added requirements matrix

$$\boldsymbol{\hat{e}\,\hat{g}^{-1}\,D(I - B_d D)^{-1}} \quad (13)$$

To compute the bilateral import requirements we must first create a diagonalized matrix of bilateral imports to output ratios by industry. For each of the bilateral import matrices M, we need to first compute total bilateral imports by industry *Mi*. From then we create a diagonalized matrix of imports to output ratio.

$$\widehat{Mi}\,\hat{g}^{-1}$$

Pre-multiplying the industry-by-commodity domestic requirements matrix by the above matrix yields the bilateral import requirements matrix

$$\boldsymbol{\widehat{Mi}\,\hat{g}^{-1}\,D(I - B_d D)^{-1}} \quad (14)$$

6. The ITA proposes that all commodities made by an industry share the same input structure. In contrast, the CTA proposes that each commodity has a unique input structure that is independent of the producing industry. For more information on the hybrid technology approach see Guo, Lawson, and Planting "From Make-Use to Symmetric I-O Tables; An Assessment of Alternative Technology Assumptions."

When utilizing value added and import requirements matrices (13) and (14) for TiVA calculations these matrices need to be post-multiplied by exports. In order to properly decompose bilateral exports into their component value added and import pieces it is helpful to transform the export vectors x into a diagonal matrix of exports $\hat{\boldsymbol{x}}$ where each diagonal entry shows the exports of a particular commodity. This diagonalized matrix of commodity bilateral exports is then post-multiplied by the value added and import requirements.

$$\hat{\boldsymbol{e}}\,\hat{\boldsymbol{g}}^{-1}\boldsymbol{D}(\boldsymbol{I}-\boldsymbol{B}_d\boldsymbol{D})^{-1}\hat{\boldsymbol{x}} \qquad (15)$$

$$\widehat{\boldsymbol{Mi}}\,\hat{\boldsymbol{g}}^{-1}\boldsymbol{D}(\boldsymbol{I}-\boldsymbol{B}_d\boldsymbol{D})^{-1}\hat{\boldsymbol{x}} \qquad (16)$$

Matrices (15) and (16) are industry-by-commodity matrices showing the value added and import content of exports. Each column of matrix (15) for example shows the value added content embedded in the exports from the corresponding column of matrix $\hat{\boldsymbol{x}}$. Likewise each column of matrix (16) shows the import content embedded in the exports from the corresponding column of matrix $\hat{\boldsymbol{x}}$. It is from the content of matrices (15) and (16), as well as statistics on exports, imports, and output that BEA's TiVA statistics are computed.

For example, to compute the domestic value added content of US exports to Canada, we would need to use a bilateral export matrix $\hat{\boldsymbol{x}}$ that includes only exports to Canada. The results from $\hat{\boldsymbol{e}}\,\hat{\boldsymbol{g}}^{-1}\boldsymbol{D}(\boldsymbol{I}-\boldsymbol{B}_d\boldsymbol{D})^{-1}\hat{\boldsymbol{x}}$ in this case would show the domestic value added content of those exports both by value added contributing industry (by row) and by commodity exported (by column). The domestic value added content of the US, when shown by contributing industry, is simply a row sum of the matrix $\hat{\boldsymbol{e}}\,\hat{\boldsymbol{g}}^{-1}\boldsymbol{D}(\boldsymbol{I}-\boldsymbol{B}_d\boldsymbol{D})^{-1}\hat{\boldsymbol{x}}$.