The Contribution of Reallocation to U.S. GDP Growth: Measurement Using Tiered Aggregation

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Date October 13, 2021

Abstract Resources moving from less productive to more productive sectors can increase aggregate output without any underlying change in production technology, yet the impact of these reallocations is challenging to measure because it involves measuring unobserved counterfactual production where resources have not moved. We construct measures of counterfactual production by implementing an Industry-Level Production Account with a tiers structure. Aggregate gross domestic product (GDP) and total factor productivity growth constructed bottom-up from the micro- (industry) level captures the true data-generating process for the sources of growth. The counterfactual accounts employ restrictions that impose a constraint that reallocating outputs and inputs have no impact on aggregate production, so that the difference between the two measures captures the economic impact of reallocations. We find that reallocations contributed 0.30 percent per year on average out of total GDP growth of 2.39 percent per year from 1987–2018. Almost all of this can be accounted for as reallocations of value added within manufacturing (for example, to the computer producing sector from other manufacturing sectors) and across sectors to the information and trade sectors.

Keywords Reallocation, Growth and Productivity, Industry Accounting

JEL codes E01

I. Introduction

The introduction of official industry-level economic accounts, such as the Integrated Industry-Level Production Account by the U.S. Bureau of Economic Analysis (BEA) and the Bureau of Labor Statistics (BLS) (Fleck et al. 2014; Garner et al. 2020) has allowed for a more refined empirical analysis of economic growth, productivity, and structural change. These data show the dispersion of industry total factor productivity (TFP) growth and factor intensities that permit a bottom-up analysis of the sources of economic growth. As a building block, these accounts are constructed to answer questions about what is happening within industries and to allow a discussion of how changes in the industry composition affect aggregate gross domestic product (GDP). There are various established methods to measure the industry origins of aggregate growth and productivity—the separate contributions of industry-level growth and reallocations across industries—but there is no official accounting method.

The main objective of this paper is to measure the economic impact of production moving across producers ("reallocations") on aggregate growth in a framework where the economic assumptions and interpretations are clearly laid out. The impacts come from two mechanisms in our simple model: the first is from capital and labor inputs moving from less to more productive sectors. When primary inputs are reallocated in this case, aggregate output would increase without any underlying change in industry-level production technology. The second mechanism involves output shifting to more productive industries due to industry-specific increases in total factor productivity (TFP). Holding inputs fixed, with an increase in TFP in the computer industry, for example, the composition of aggregate output shifts toward computer production even though no inputs have moved. We label the impact of shifting the composition of output as output reallocation to describe how changes in the structure of the economy contribute to aggregate GDP growth.

The basic intuition for our approach to measuring the impact of reallocations is that we compare the sources of economic growth under the observed data generating process where resources are free to move between sectors to a counterfactual version of the data that imposes the assumption that there is no economic impact from resource movements across sectors. One may think of this as comparing the accounting under industry-specific functions versus the accounting under an imposed common function. Our method uses a tiered structure of production where we are able to assess the impact of reallocations across micro units (industries) within major sectors (for example, across paper and computer in the manufacturing group), and across major sectors (for example, between manufacturing and services).

A second purpose of this paper is to discuss different schemes for aggregating micro units to account for the sources of GDP growth. We codify the economic assumptions that are embedded in each aggregation scheme, and how these assumptions impact the reallocations.

One can illustrate the intuition for our approach with a simple example of using the aggregate GDP accounts. One often sees GDP (Y) written as a linear sum of final demand (say C and I), and equal to a function of capital (K), labor (L) and TFP (T):

$$Y = C + I = F(K, L, T)$$

First, note that the first equality is not an accounting identity; the accounting identity for this is the equation in nominal values where nominal gross output equals the value of consumption plus investment goods, and this equals nominal factor payments. Using p's to denote prices, these identities are:

$$p_Y Y = p_C C + p_I I = p_K K + p_L L$$

On the other hand, the statement that $Y = C + I$ imposes the assumption that reallocating a unit of production from consumption to investment has no impact on aggregate GDP. A general way of expressing this without *a priori* restrictions on reallocations is *Y=g(C,I)*.

This example also demonstrates how we measure our reallocation estimates. We compare the sources of GDP growth measured under production technologies that rule out reallocations *a priori* to alternative measures that embed the reallocations. The difference between the two is the contribution of reallocation to economic growth. This basic way of conceptualizing reallocations is not new; it was employed in Jorgenson (1988) to study postwar productivity growth in the United States. Samuels (2017) demonstrates that it is important to account for heterogeneous inputs when measuring the impact of reallocations, so we use the same input classification as used in that paper. Corrado, Haskel, and Jona-Lasini (2019) uses a similar method to study capital reallocation across countries during the Great Recession. Our main innovation is to introduce tier accounting that allows us to trace reallocations of outputs and inputs on growth and TFP both within and across sectors. We make use of a novel U.S. dataset that merges official industry-level production data with detailed data on input use across industries that controls for skill mix and capital asset composition across sectors. We demonstrate that under one particular aggregation structure (industries to sectors to GDP), sector TFP growth can be measured using standard growth accounting indexes where sector outputs and inputs are constructed with particular aggregator functions over the component industries. This informs how industry data can be used to construct sector data in a consistent way. Sector accounts often provide a more digestible view than growth accounting for many detailed industries.

There are other methods used to discuss the effects or contributions of structural change, but they involve the use of counterfactual parameters. For example, the shift share method is used to calculate the contributions of within-industry effects and structural change effects for labor productivity.[1] Mas et al.

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[1] The shift-share method is used, for example, in Mas, Milana, and Serrano (2012) to compare changes over time as well as comparing different economies, and in Hofman et al. (2016) to discuss Latin American economic development.

(2012) gives the following decomposition for aggregate value added (VA) per unit labor (Y_t / L_t) in terms of industry productivities (Y_{jt} / L_{jt}):

$$\frac{Y_t}{L_t} - \frac{Y_0}{L_0} = \sum_j \theta_{j0} \left(\frac{Y_{jt}}{L_{jt}} - \frac{Y_{j0}}{L_{j0}} \right) + \sum_j (\theta_{jt} - \theta_{j0}) \frac{Y_{jt}}{L_{jt}}$$

where θ_{jt} is industry j's share of total labor at time t. The first term on the right is the within-industry effect and the second term is the structural change effect. This is a convenient decomposition used by many authors, but it has a disadvantage of not distinguishing the capital and labor flows, does not identify the role of TFP growth within industries, and assumes that structural change is one that is the same as the change in value-added shares of industries. To see why this approach potentially obscures useful differences between reallocation and other "structural change," consider an economy where the farm sector shrinks from 20 percent of nominal GDP to 5 percent. This could occur because farm prices are falling relative to other prices (and thus not really due to resource reallocation) or farm output is falling relative to other types of output.

A slightly different formulation is given in Oulton (2020), including the following general formula for the change in aggregate labor productivity ($Z_t = V_t / L_t$) in terms of industry productivities (Z_{jt}):

$$\frac{Z_t}{Z_{t-1}} = \sum_j \frac{v_{jt-1}}{w_{jt-1}} w_{jt} \frac{Z_{jt}}{Z_{jt-1}}$$

The v's are the value-added shares and w's are labor shares. This general form of aggregate labor productivity growth is then compared to the case where labor shares are held fixed ($\frac{Z_t}{Z_{t-1}} = \sum_j v_{jt-1} \frac{Z_{jt}}{Z_{jt-1}}$), and the case where all industries have the same initial productivities ($\frac{Z_t}{Z_{t-1}} = \sum_j w_{jt} \frac{Z_{jt}}{Z_{jt-1}}$). The gaps between these three growth paths give a measure of the importance of structural change. This is also a convenient calculation but involves the use of counterfactual parameters.

An advantage of our approach in comparison to such alternatives is that we do not use counterfactual constant shares. Our method compares aggregate GDP results under alternative aggregation approaches, those that embed reallocation effects to an alternative set of aggregation schemes where reallocation effects are limited or ruled out by assumption. We use different levels of detail in computing average prices and factor costs but do not impose counterfactual values. Because the long and established literature on index numbers takes nominal values as the starting point for constructing real index measures (often involving important economic assumptions), we intentionally avoid changing observed nominal transactions which (if measured correctly) represent true economic history and we have no basis for changing. Furthermore, our method also accounts for capital, labor, and TFP separately to show the impact of each on aggregate reallocation.

We find that between 1987 and 2018 output reallocations accounted for a significant share of aggregate GDP growth (0.30 percentage points out of total GDP growth of 2.39 percent per year). Over this same period, we find that the impact of input reallocations was mostly negligible; an implication of this is that TFP growth is mismeasured to be too high by about 0.30 percent per year if one does not allow for reallocations in the specification of aggregate production technology. Over medium-term cycles, we do find some evidence that input reallocations are important. For example, in the 1987–1995 period, capital input reallocations contributed about 0.13 percentage point to aggregate GDP growth (of 2.45 percent per year) and the impact of labor reallocations was slightly negative. In the sections below, we provide evidence on the industry and sector origins of the aggregate reallocation contributions.

II. Methodology and Results

II.1 Industry accounts and aggregation over industries

We start by modeling the aggregate economy as a collection of industries by specifying industry production functions and an aggregator function. The important feature of our aggregation function is that it permits substitution of output and inputs to impact GDP. We take this initial specification as the true data generating process for industry and aggregate economic growth, that is, one with the less restrictive assumptions to represent the true sources of economic growth.

For each basic industry *j* we have the familiar KLEMS accounting[2] and express output at time *t* as a Tornqvist index of capital, labor, intermediate inputs, and the level of technology, T_{jt}:

$$\begin{aligned}
&\Delta \ln Q_j = \overline{w_{Kj}}\Delta \ln K_j + \overline{w_{Lj}}\Delta \ln L_j + \overline{w_{Xj}}\Delta \ln X_j + \Delta \ln T_j \\
&\Delta \ln Q_{jt} = \ln Q_{jt} - \ln Q_{j,t-1} \\
&P_j^Q Q_j = P_j^K K_j + P_j^L L_j + P_j^X X_j \\
&w_{Kj} = \frac{P_j^K K_j}{P_j^Q Q_j}\text{; etc. for L,X} \\
&\overline{w_{Kjt}} = \tfrac{1}{2}(w_{Kjt} + w_{Kj,t-1})
\end{aligned} \tag{1}$$

The P variables denote prices and w's the value shares. *X* denotes total intermediate input, aggregating over intermediate purchases of commodities. In the third line in equation (1) the revenue equals total cost assumption where the (*ex post*) cost of capital is the residual. P_j^Q is the producer price (seller's price) for the output of j.

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[2] The KLEMS framework is well-established, so we shall be brief here. Readers may consult Jorgenson, Ho, and Stiroh (2005) for a more detailed description.

Importantly for our reallocation estimates, industry capital input is an aggregate over asset types:

$$\begin{aligned} \Delta \ln K_j &= \sum_a \overline{w_{aj}^K} \Delta \ln K_{aj} ;\text{a,k=1,…,100 asset types} \\ w_{aj}^K &= \frac{P_{aj}^K K_{aj}}{\sum_k P_{kj}^K K_{kj}} \\ P_j^K K_j &= \sum_a P_{aj}^K K_{aj} \end{aligned} \tag{2}$$

Similarly, labor input is an aggregate over different heterogenous labor types within an industry:

$$\begin{aligned} \Delta \ln L_j &= \sum_l \overline{w_{lj}^L} \Delta \ln L_{lj} ;\text{l,m=1,…,84 labor types} \\ w_{lj}^L &= \frac{P_{lj}^L L_{lj}}{\sum_m P_{mj}^L L_{mj}} \\ P_j^L L_j &= \sum_l P_{lj}^L L_{lj} \end{aligned} \tag{3}$$

Intermediate inputs are given by the Use matrix (U_{ij}); the nominal value of commodity input *i* into industry *j* is $U_{ij} = P_i^C X_{ij}$. Writing in this way means we are assuming all industries pay the common price P_i^C for commodity *i* (that is, ignoring more detailed information below the *i* classification). The aggregate intermediate input into *j* is an index over all commodities:

$$\begin{aligned} \Delta \ln X_j &= \sum_i \overline{w_{ij}^X} \Delta \ln X_{ij} \\ w_{ij}^X &= \frac{P_i^C X_{ij}}{\sum_k P_k^C X_{kj}} \\ P_j^X X_j &= \sum_i P_i^C X_{ij} \text{i,k=1,…,63 commodities} \end{aligned} \tag{4}$$

With the above pieces we can now define real value added, V_j, using the double deflation method, which becomes important in assessing the impact of industries and sectors in determining reallocation. We first express gross output as an index of value added and intermediate inputs:

$$\Delta \ln Q_j = \overline{w_{Vj}} \Delta \ln V_j + \overline{w_{Xj}} \Delta \ln X_j \tag{5}$$

which implicitly defines real value-added growth for industry j using the growth rate of real industry output and intermediate input and the corresponding nominal shares. Importantly, this is the method used in the official BEA-BLS Integrated Industry-Level Production Account.[3]

Aggregation

Our aggregation function for our baseline model is the flexible production possibility frontier (PPF) of Christensen, Jorgenson, and Lau (1973) and we refer readers to the subsequent literature that discusses the properties of translog aggregation functions that are referenced in Jorgenson, Ho, and Stiroh (2005). The key property of the translog function for our application is that it permits substitutions in outputs and inputs to affect aggregate GDP. By assuming this functional form for aggregate GDP, the industry sources of aggregate GDP are obtained as in Jorgenson, Ho, and Stiroh (2005, ch. 8). The key equations are:

$$\Delta \ln V_{AI} = \sum_j \overline{w_j} \Delta \ln V_j \tag{6}$$

$$\Delta \ln V_{AI} = \sum_j \overline{w_j} \frac{\overline{w}_{K,j}}{\overline{w}_{V,j}} \Delta \ln Q_{Kj} + \overline{w_j} \frac{\overline{w}_{L,j}}{\overline{w}_{V,j}} \Delta \ln Q_{Lj} + \overline{w_j} \frac{1}{\overline{w}_{V,j}} \Delta \ln T_j \tag{7}$$

Equation (6) gives aggregate real value-added growth (GDP at factor cost) as a Tornqvist index over industry value added where the AI subscript denote "aggregate over industry." Equation (7) comes from substituting the first line of (1) into (6) and gives contributions of industry capital, labor, and TFP growth to aggregate real value-added growth.

II.2 Sector aggregates and within-sector reallocations

To assess the role of reallocations among industries within sectors (think within manufacturing and within services), we specify and implement an alternative model that is based on sector-level aggregates. The basic intuition for this construction is that it explicitly ignores differences across component industries by imposing homogeneity within each sector in our sector definitions. The is analogous to the example given for the aggregation over investment and consumption goods to give demand-side GDP in the introduction. Within sectors, we impose the assumption that output is homogenous (for example, computers are perfectly substitutable with paper) and thus rule out reallocations that impact sector-level output. The same applies for capital and labor inputs; moving a unit of capital or labor across industries within as sector leaves sector-level input (and thus its contribution to output) unchanged.

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[3] Integrated Industry-Level Production Account (KLEMS) | U.S. Bureau of Economic Analysis (BEA)

To limit the number of equations we note that the sector version of the accounts can be written with the same equations as the industry accounts above, except that instead of indexing over j industries, we index over s sectors. While the sector equations are very similar to the industry version, the construction of the output and input indexes must be adjusted to conform to the economic assumption that substitution between industries does not impact sector-level growth.

To produce model-consistent data, our approach assumes there is no industry-level information on inputs, all component industries of sector *s* pay the same price for an input (a given type of worker, a given asset class, or a given intermediate commodity). This creates a production function for *s* based on intermediate inputs identified only for S sectors. We first create nominal input-output tables (Use and Make tables) for the S sectors by summing over the values of the industries that comprise sector s. We then create output and commodity prices at the sector level. Following Jorgenson, Ho, and Stiroh (2005) terminology, we call this a production function (PF) approach, as opposed to the PPF approach in equation (6) that allows each industry to have its own price. By this common price aggregation (or production function) approach, real gross output of s is given as a simple linear sum of the output of component industries:

$$Q_s^{SS} = \sum_{j \in s} Q_j \tag{8}$$

As in the investment and consumption example, it is straightforward to observe that reallocating a unit of output within sectors has no impact on sector-level output. The SS superscript denotes "simple sum" to distinguish this from a measure of sector output defined as a Tornqvist index over industry output (discussed in the appendix). The sector output price, $P_s^{Q;SS}$, is given by deflating the nominal sector output by Q_S^{SS}; this reflects the assumption that all outputs within a sector receive the same price: The commodity output and prices are given by a Make matrix of dimension S that is aggregated over the industry-level Make matrix in a similar way.

We express the output of s at time t as a function of the capital, labor, intermediate inputs and TFP in a way analogous to (1):

$$Q_{st}^{PF} = Q_{st}^{SS} = f(K_{st}, L_{st}, X_{st}, T_{st}^{PF}); \text{ s=1,…,S} \tag{9}$$

At the basic 63-industry level, X_{ij} denotes the real intermediate input of commodity i into industry j (equation (4)), and P_i^C is its price. For the more aggregated S×S Use table, let X_{rs}^S denote the real intermediate input *r* into sector s. This is not given as an index over the *j* industries' intermediate inputs, but is simply the total nominal input (adding over X_{ij}) divided by the commodity price of r:

$$X_{rs}^S = \frac{\sum_{i \in r} \sum_{j \in s} P_i^C X_{ij}}{P_r^{C;S}}; \text{ r,s=1,…,S} \tag{10}$$

The aggregate intermediate input into industry s is the Tornqvist index of all commodities r:

$$\Delta \ln X_s^S = \sum_r \overline{w_{rs}^{X;S}} \Delta \ln X_{rs}^S$$
$$w_{rs}^{X;S} = \frac{P_r^{C;S} X_{rs}^S}{\sum_q P_q^{C;S} X_{qs}^S} \tag{11}$$

The price of the intermediate bundle, $P_s^{X;S}$, is given by dividing the nominal total by the aggregate input index: $P_s^{X;S} = \sum_q P_q^{C;S} X_{qs}^S / X_s^S$.

The capital input into sector *s* (K_s^S) is not derived by an index over the capital input of each component industry but by imposing the constraint that all components earn the average rate of return in *s*. That is, it is constructed by first calculating the stock of capital of *s* (A_s^S) by summing over all component industries (for each asset type a):

$$A_{as}^S = \sum_{j \in s} A_{aj} \tag{12}$$

The total capital input into sector *s* is the Tornqvist index over all the asset types where the cost of capital for type *a* is $P_{as}^{K;S}$:

$$\Delta \ln K_s^S = \sum_a \overline{v_{as}^{K;S}} \Delta \ln A_{as}^S$$
$$v_{as}^{K;S} = \frac{P_{as}^{K;S} A_{as}^S}{\sum_k P_{ks}^{K;S} A_{ks}^S} \text{;a,k=1,2,… 100 asset types} \tag{13}$$

The price of aggregate capital input for *s*, $P_s^{K;S}$, is given by dividing the total nominal value by the quantity from (13): $P_s^{K;S} = \sum_k P_{ks}^{K;S} A_{ks}^S / K_s^S$

Labor input into sector s is similarly constructed by imposing the constraint that all workers of type *l* in *s* earn the same average wage. Labor input at the most basic level is the annual hours worked by type *l* in industry *j*. The input at the sector level is thus the simple sum of hours assuming that each hour in all the component industries earn the same wage:

$$H_{ls}^S = \sum_{j \in s} H_{lj} \text{;l=}\{\text{sex,age,educ,class}\} \tag{14}$$

Effective labor input is an aggregate over hours worked by all types:

$$\begin{aligned} \Delta \ln L_s^S &= \sum_l \overline{v_{ls}^{L;S}} \Delta \ln H_{ls}^S \\ v_{ls}^{L;S} &= \frac{P_{ls}^{L;S} H_{ls}^S}{\sum_k P_{ks}^{L;S} H_{ks}^S} \end{aligned} \tag{15}$$

The price of aggregate labor input for *s*, $P_s^{L;S}$, is given by dividing the total nominal value by the quantity (15).

We now have all the pieces to calculate TFP growth at the sector level. Gross output of sector *s* is a Tornqvist index of the inputs and TFP, T_s^{PF}:

$$\begin{aligned} \Delta \ln Q_s^{PF} &= \overline{w_{Ks}^S} \Delta \ln K_s^S + \overline{w_{Ls}^S} \Delta \ln L_s^S + \overline{w_{Xs}^S} \Delta \ln X_s^S + \Delta \ln T_s^{PF} \\ w_{Ks}^S &= \frac{P_s^{K;S} K_s^S}{P_s^{Q;S} Q_s^{PF}} \end{aligned} \tag{16}$$

The base year units are chosen to satisfy the value equation for output and total nominal cost of inputs at prices normalized to 1:

$$P_s^{Q;S} Q_s^{PF} = P_s^{K;S} K_s^S + P_s^{L;S} L_s^S + P_s^{X;S} X_s^S \tag{17}$$

The real value added of sector s, V_s^{PF}, is output less aggregate intermediate input:

$$\begin{aligned} \Delta \ln Q_s^{PF} &= \overline{w_{Vs}^S} \Delta \ln V_s^{PF} + \overline{w_{Xs}^S} \Delta \ln X_s^S \\ \overline{w_{Vs}^S} \Delta \ln V_s^{PF} &= \overline{w_{Ks}^S} \Delta \ln K_s^S + \overline{w_{Ls}^S} \Delta \ln L_s^S + \Delta \ln T_s^{PF} \\ P_s^{V;S} V_s^{PF} &= P_s^{K;S} K_s^S + P_s^{L;S} L_s^S \end{aligned} \tag{18}$$

Given these sector-level accounts, we aggregate to total GDP in a way analogous to the industry-level approach in (6) and (7), where we now use an AS subscript to denote "aggregate over sectors":

$$\begin{aligned} \Delta \ln V_{AS}^{SS} &= \sum_s \overline{w_s} \Delta \ln V_s^{PF} \\ \Delta \ln V_{AS}^{SS} &= \sum_s \overline{w_s} \frac{\overline{w_{Ks}^S}}{\overline{w_{Vs}^S}} \Delta \ln K_s^S + \overline{w_s} \frac{\overline{w_{Ls}^S}}{\overline{w_{Vs}^S}} \Delta \ln L_s^S + \overline{w_s} \frac{1}{\overline{w_{Vs}^S}} \Delta \ln T_s^{PF} \end{aligned} \tag{19}$$

By comparing the sources of aggregate growth under the assumption of industry-level production (7) and aggregation to sector-level production (19) where the aggregation function imposes the condition that reallocation has no impacts, we can measure the impact of reallocation on aggregate economic growth. The difference between the two GDPs yields our definition of the reallocation of value added and inputs:

$$
\begin{aligned}
&\Delta \ln V_{AI} - \Delta \ln V_{AS}^{SS} && : \text{ReallVA\_AI\_ASss} \\
&= \sum_j \overline{w_j} \frac{\overline{w}_{K,j}}{\overline{w}_{V,j}} \Delta \ln K_j - \sum_s \overline{w_s} \frac{\overline{w_{Ks}^S}}{\overline{\overline{w_{Vs}^S}}} \Delta \ln K_s^S && : \text{ReallK\_AI\_ASss} \\
&\quad + \sum_j \overline{w_j} \frac{\overline{w}_{L,j}}{\overline{w}_{V,j}} \Delta \ln L_j - \sum_s \overline{w_s} \frac{\overline{w_{Ls}^S}}{\overline{\overline{w_{Vs}^S}}} \Delta \ln L_s^S && : \text{ReallL\_AI\_ASss} \\
&\quad + \sum_j \overline{w_j} \frac{1}{\overline{w}_{V,j}} \Delta \ln T_j - \sum_s \overline{w_s} \frac{1}{\overline{\overline{w_{Vs}^S}}} \Delta \ln T_s^{PF} && : \text{ReallT\_AI\_ASss}
\end{aligned}
\tag{20}
$$

We denote the left-hand side, $\Delta \ln V_{AI} - \Delta \ln V_{ASss}$, by ReallVA_AIss to emphasize that it is the reallocation of value added from the difference between the aggregation over industry and aggregation over sectors using the simple sum method. It captures the contribution of reallocation of value added within sectors to aggregate GDP growth. To understand the later decompositions and to provide additional intuition, it is worth focusing on the industry origins of the aggregate value-added reallocation effect.

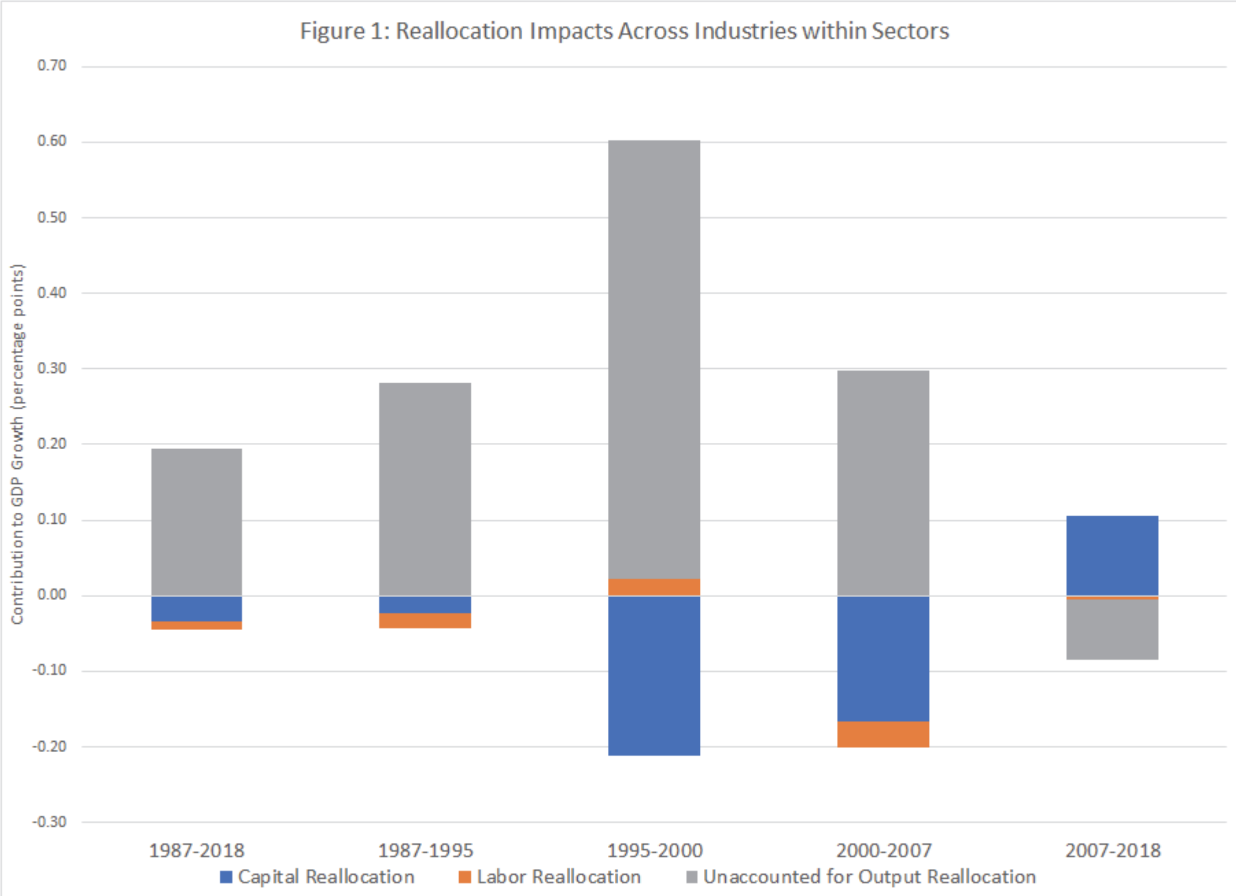
$$
\begin{aligned}
\Delta \ln V_{AI} - \Delta \ln V_{AS}^{SS} &= \sum_j \overline{w_j} \Delta \ln V_j - \sum_s \overline{w_s} \Delta \ln V_s^{PF} \\
&= \sum_s \sum_{j \in s} \overline{w_j} \Delta \ln V_j - \sum_s \overline{w_s} \Delta \ln V_s^{PF}
\end{aligned}
\tag{21}
$$

By examining (21), (18), and (5) we can see that the origins of the aggregate value-added reallocation effect involves the growth of output at the industry versus the sector level, as well as the growth of intermediate input (which depends both on structure of the use table and the assumptions used to assemble the commodity prices used to deflate the intermediate inputs). Under industry aggregation, output and intermediate inputs are "flexibly" aggregated; that is, under this structure, both outputs and intermediate inputs have moved to the most productive industry. With the sector specification, moving outputs and intermediate inputs across industries within sectors does not impact aggregate VA growth by assumption. Therefore, the difference between industry-based and sector-based aggregation captures the impact of reallocation on aggregate GDP growth.

An advantage of our approach is that it allows us to decompose aggregate reallocation to reallocations of capital and labor. $\sum_j \overline{w_j} \frac{\overline{w}_{K,j}}{\overline{w}_{V,j}} \Delta \ln K_j$ is the aggregate contribution of capital input to GDP growth under industry aggregation while $\sum_s \overline{w_s} \frac{\overline{w}_{K,s}}{\overline{w}_{V,s}} \Delta \ln K_s^S$ is the corresponding contribution under the assumption that reallocating an input within a sector has no impact on sector-level production; thus, the difference between the two (ReallK_AI_ASss) represents the contribution of reallocation to the aggregate contribution of capital input. The same interpretation holds for the labor input terms.

The interpretation of the TFP terms is a bit different. $\sum_j \overline{w_j} \frac{1}{\overline{w}_{V,j}} \Delta \ln T_j$ is the sum of the contributions of each industry's TFP to aggregate TFP growth. In our specifications, this is considered true aggregate TFP growth. $\sum_s \overline{w_s} \frac{1}{\overline{w}_{V,s}} \Delta \ln T_s^{PF}$ is the contribution of each sector to aggregate TFP growth under the (wrong) assumption that outputs and inputs are homogenous within each sector. Thus, the difference between the two represents the impact of mismeasuring sector-level TFP instead of true TFP on aggregate TFP and GDP growth. Note that the difference between these two terms (ReallT_AI_ASss) reflects the difference between the value-added reallocation and capital and labor input reallocations. Furthermore, the value-added reallocation reflects industry gross output reallocations and industry intermediate input reallocations. Thus, the TFP reallocation effect can be interpreted as value-added reallocation that is unaccounted for by the reallocation of capital and labor input reallocations. The intuition for this is output and intermediate inputs moved to industries where they are relatively more productive without a corresponding shift in capital and labor services.

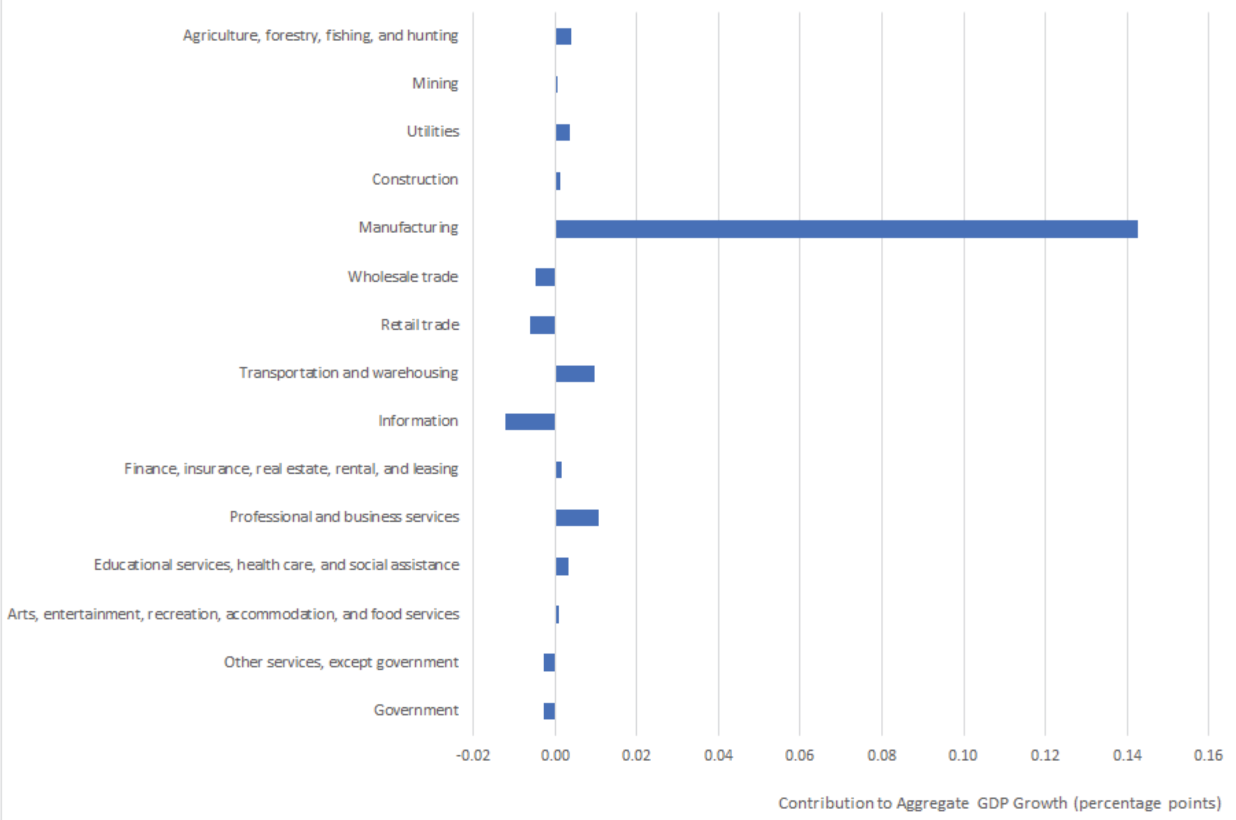
In the figures below, ReallK_AI_ASss is the contribution of the reallocation of capital, ReallL_AI_ASss is for the reallocation of labor and ReallT_AI_ASss is the "unaccounted for output reallocation." Finally, note that by summing over each j within each sector s, we can determine the sector-level contributions of each reallocation term.



II.2.2 Results of industry versus sector aggregation

We plot the three reallocation terms on the right-hand side of (20) in the bar graphs in figure 1, the first bar for the entire period 1987–2018 and the next three bars for the subperiods 1987–1995, 1995–2000, 2000–2007, and 2007–2018. These subperiods correspond to the "long slump," "investment boom," "jobless recovery," and "great recession and recovery" periods in Jorgenson, Ho and Samuels (2019). Each bar consists of the three components—the contributions of reallocation of capital, labor, and "unaccounted for output reallocation" to GDP growth. Figure 1 shows that reallocations within sectors accounted for a significant share of aggregate GDP growth over the 1987–2018 period (0.15 percentage points out of total GDP growth of 2.39 percent per year) and over the subperiods considered. Over the whole period, the contribution of capital and labor reallocations were minor; thus, shifts in output across industries within sectors that were unaccompanied by corresponding shifts in capital and labor accounted for the majority of the total reallocation impacts. An alternative interpretation of this "unaccounted" impact is the impact of mismeasured TFP using sector data that subsumes the underlying industries. That is, using sector-only data would lead to an overestimate of the contribution of TFP to aggregate GDP growth of about 0.20 percentage points per year over the 1987–2018 period.

Figure 2: Within Sector Origins of Reallocation Impacts 1987-2018



The results show that the pattern of reallocation changed across subperiods. The 1987–1995 subperiod looked like the period as a whole. During the information technology investment boom of 1995–2000 there was a very large contribution of reallocation to GDP growth (0.39 percentage points out of 4.15 percent per year) and the contribution of capital reallocations was negative over this period. That signifies that as output was moving toward relatively more productive industries within sectors, capital input moved towards relatively less productive sectors. During the 2000–2007 period, both capital and labor reallocations were negative (though the labor reallocation was relatively small), so the net impact of reallocation over this period was relatively small. Finally, during the Great Recession and recovery period, the net reallocation was approximately zero, while the contribution of capital reallocation was slightly positive, the output reallocation unaccounted by input movement was negative during this period in contrast to the other periods in the sample. This indicates that during 2007–2018, output within sectors was actually moving toward relatively unproductive industries.

Next, we plot the sector-specific terms on the right-hand side of (20); that is, we compute the capital reallocation contribution for sector s as $\sum_{j\in s} \overline{w}_j \frac{\overline{w}_{K,j}}{\overline{w}_{V,j}} \Delta \ln K_j - \overline{w}_s \frac{\overline{w^S_{Ks}}}{\overline{w^S_{Vs}}} \Delta \ln K^S_s$, and similarly for the labor and TFP reallocations. Figure 2 shows the sum of the capital, labor, and unaccounted output/TFP mismeasurement contributions for each of the 15 sectors. This gives the sector origins of the reallocation impacts for the period as a whole (for the total output reallocation effect, equal to the sum of the contributions of the capital, labor, and unaccounted for output effect). This decomposition shows that the preponderance of the economy-total output reallocation effect manifested from the manufacturing sector, likely reflecting the impact of the changes in computer and electronic products industry within manufacturing.

II.3 Sector aggregates and reallocation across sectors

In this section, we assess the impact of reallocations across sectors on GDP growth (for example, from manufacturing to services or from farm to manufacturing). The starting point is the sector accounts described in section II.2. The point of comparison is a production technology where aggregate reallocation effects across sectors is constrained to be zero. That production technology corresponds to the aggregate production function of Solow (1957). Under this specification, substituting output or inputs across industries or sectors has no impact on aggregate GDP by construction. The strong assumptions of the aggregate production function requires that prices for inputs are the same across all industries and outputs are perfectly substitutable. This implies that (starting from sectors) the aggregate value-added quantity index is defined as:

$$V_S^{APF} = \sum_s V_s \tag{22}$$

where the APF superscript denotes aggregate production function, as opposed to the PPF in equation (19).

This states that aggregate GDP(APF) is defined simply as the sum of the quantity indexes of value added for each sector (under the restriction that the price of value added for each industry is the same). To reiterate, this amounts to imposing no GDP impact from reallocating output across sectors. The difference between the GDP given in (19) by a Tornqvist aggregation over “simple sum” sector value added, and the GDP(APF) given in (22) as a linear sum of sector value added yields the value added reallocation terms $\Delta \ln V_{AS}^{SS} - \Delta \ln V_s^{APF}$. To illuminate the sector origins of this aggregate reallocation effect, we use an approximation to compare the sources of growth under sector aggregation to the sources of growth under the assumptions of the aggregate production function. These reallocation effects are defined by the following equation:

$$\Delta \ln V_{AS}^{SS} - \Delta \ln V_s^{APF} =$$
$$(\sum_s \overline{w_s} \frac{\overline{w}_{K,s}}{\overline{w}_{V,s}} \Delta \ln K_s - \overline{w_{KAPF}} \Delta \ln K_{APF}) + (\sum_s \overline{w_s} \frac{\overline{w}_{L,s}}{\overline{w}_{V,s}} \Delta \ln L_s - \overline{w_{LAPF}} \Delta \ln L_{APF}) \tag{23}$$
$$+ \sum_i \overline{w_s} \frac{1}{\overline{w}_{V,s}} \Delta \ln T_s^{PF} - \Delta \ln T_S^{APF}$$

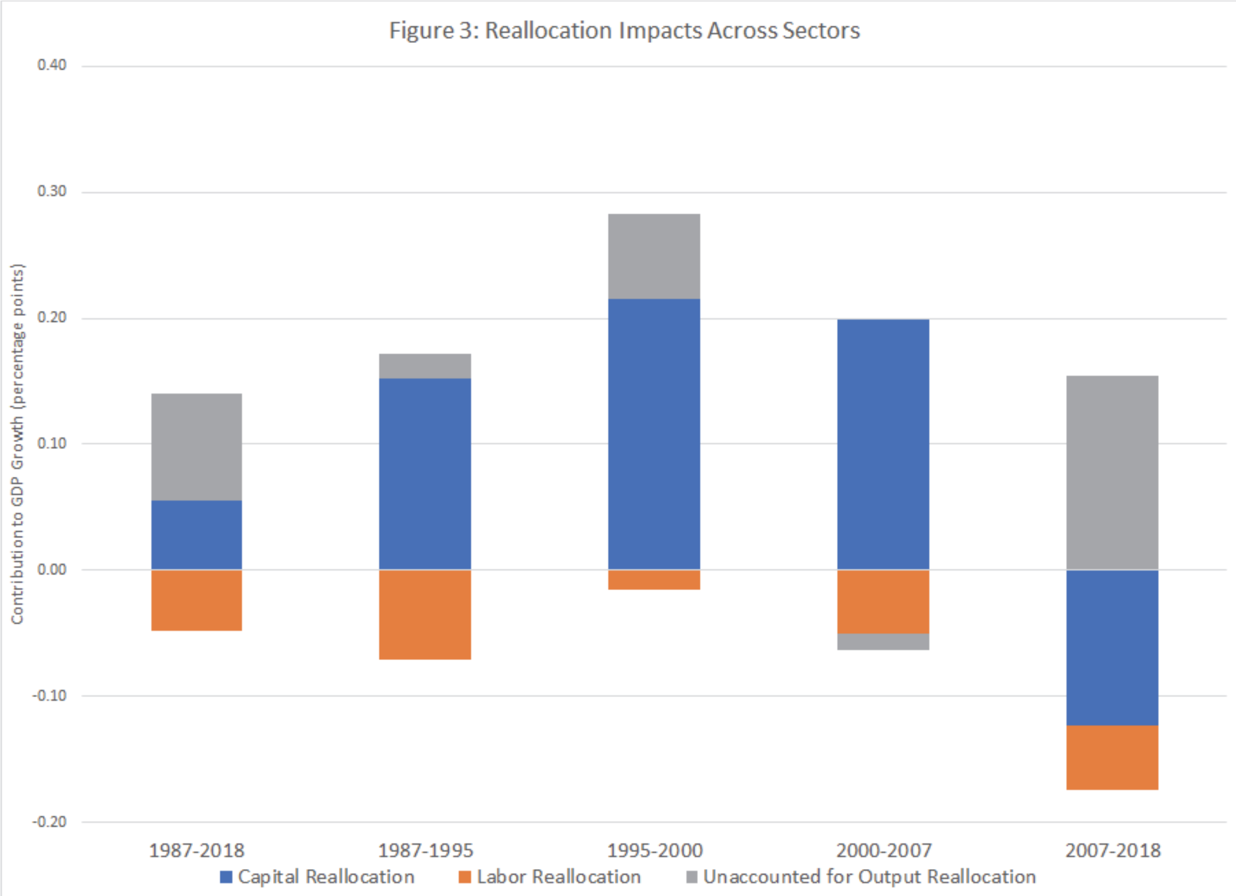
We approximate this at the sector level by redefining value added, capital, and labor at the sector level by imposing the implicit price indexes from the aggregate production function at the sector level. We label these terms: V_s^{APF}, K_{APF}, and L_{APF}, which represent the quantity indexes at the sector level computed using the aggregate prices. We use these quantities to construct the aggregate sources of growth, yielding the following decomposition of reallocations:[4]

$$\Delta \ln V_{AS}^{SS} - \Delta \ln V_s^{APF} =$$
$$\sum_s \overline{w_s} \Delta \ln V_s^{PF} - \sum_s \overline{w_s} \Delta \ln V_s^{APF} \quad : \text{REALVA\_ASss\_APFSS}$$
$$= \sum_s \overline{w_s} \frac{\overline{w}_{K,s}}{\overline{w}_{V,s}} \Delta \ln K_s - \sum_s \overline{w_i} \frac{\overline{w}_{K,s}}{\overline{w}_{V,s}} \Delta \ln K_s^{APF} : \text{REALK\_ASss\_APFSS} \tag{24}$$
$$+ \sum_s \overline{w_s} \frac{\overline{w}_{L,s}}{\overline{w}_{V,s}} \Delta \ln L_s - \sum_s \overline{w_s} \frac{\overline{w}_{L,s}}{\overline{w}_{V,s}} \Delta \ln L_s^{APF} : \text{REALL\_ASss\_APFSS}$$
$$+ \sum_i \overline{w_s} \frac{1}{\overline{w}_{V,s}} \Delta \ln T_s^{PF} - \sum_s \overline{w_s} \frac{1}{\overline{w}_{V,s}} \Delta \ln T_s^{APF} : \text{REALT\_\_ASss\_APFSS}$$

This decomposition gives us the contributions of reallocations across sectors by comparing the sources of growth allowing for sector reallocation effects (from the previous section) to the aggregate production function that rules out sector reallocation effects.

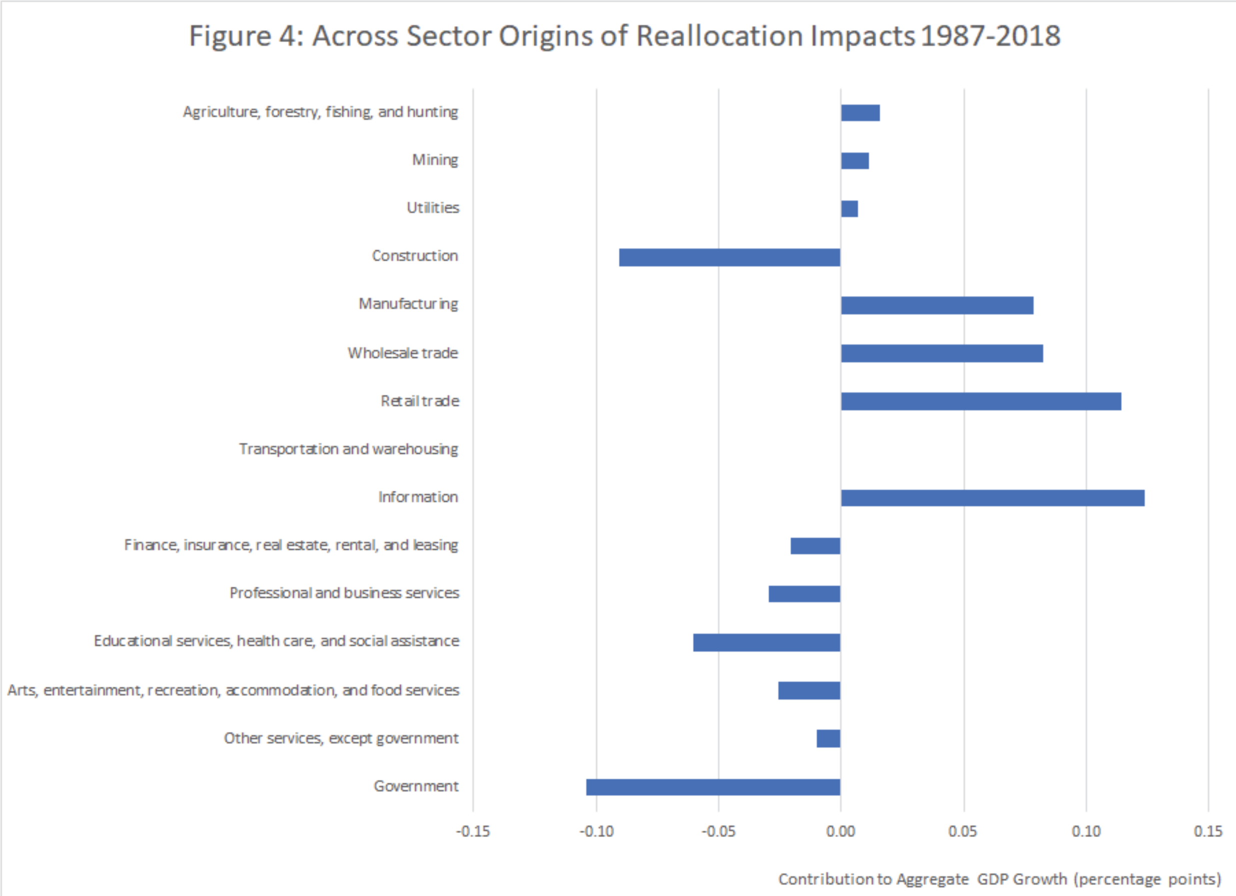
---

[4] We approximate the growth of V_S^{APF} as $\Delta \ln V_S^{APF} = \Delta \ln \sum_s V_s \approx \sum_s \overline{w}_s \Delta \ln V_s^{APF}$ where $V_s^{APF} = P_s^V V_s \ / \ P_V^{APF}$ is the "aggregate production function" value added of sector s defined as the nominal value added of s divided by a common, aggregate, price of value added (P_V^{APF}). That is, $\Delta \ln(x+y) \approx \text{tornqvist}(x+y)$ when the common price is imposed.



Results

We plot the three terms on the right-hand side of (24) in figure 3—capital reallocation contribution, labor reallocation, and "unaccounted for" reallocation—for the whole 1987–2018 period and for the three subperiods. This shows that the magnitude of reallocation impacts of resources moving across sectors was somewhat smaller than the impacts of resources moving within sectors shown in figure 1 and the pattern is a bit different as well. Over the 1987–2018 period, resources moving across sectors accounted for 0.09 percentage points out of total GDP growth of 2.39 percent per year (0.06 for K, –0.05 for L and 0.08 unaccounted). In contrast to the reallocation impacts within sectors, input reallocation played a more prominent role across sectors. In particular, in each of the 1987–1995, 1995–2000, and 2000–2007 subperiods, capital reallocations made a significant positive contribution to aggregate GDP growth. In figure 1, capital reallocations were only substantial for the post 1995 period. Across sectors, labor reallocations were negative for each of the subperiods. Again, this indicates that labor moving to relatively unproductive sectors dampened GDP growth over the period.



As in figure 2, we give the contributions for each sector and plot the results in figure 4. This shows that the sector origins of the across-sector reallocation effects were dispersed across the 15 sectors (unlike the within-sector effects presented in figure 2). Resource movements involving the manufacturing, wholesale, retail, and information sectors contributed a significant boost to aggregate GDP growth while resource movements involving construction; education, health care, and social assistance; and government dampened the aggregate reallocation impact. These movements may be into or out of these sectors; that is, the positive impact attributed to Information may reflect outputs and inputs moving *into* a relatively more productive information sector or *out of* a relatively unproductive information sector to relatively more productive sectors.

II.4 Aggregate production and intermediate input reallocations

We are now ready to discuss the final piece that goes into aggregate GDP given our tier structure of industries and sectors. We started from the least restrictive aggregation method, a Tornqvist index over industry value added giving V_{AI}, in equation (7). This allowed each industry to have its own price of value added (and prices of capital, labor, and intermediate inputs). The first restrictions we placed were on the industries within a given sector; we assumed that all $j \in s$ pay the same price for inputs, and the outputs are perfectly substitutable where we write Q_s as a simple sum of component gross output. This gives us the GDP defined as V_{AS}^{SS} in equation (19). The difference between these two GDPs gives us the reallocations in (20).

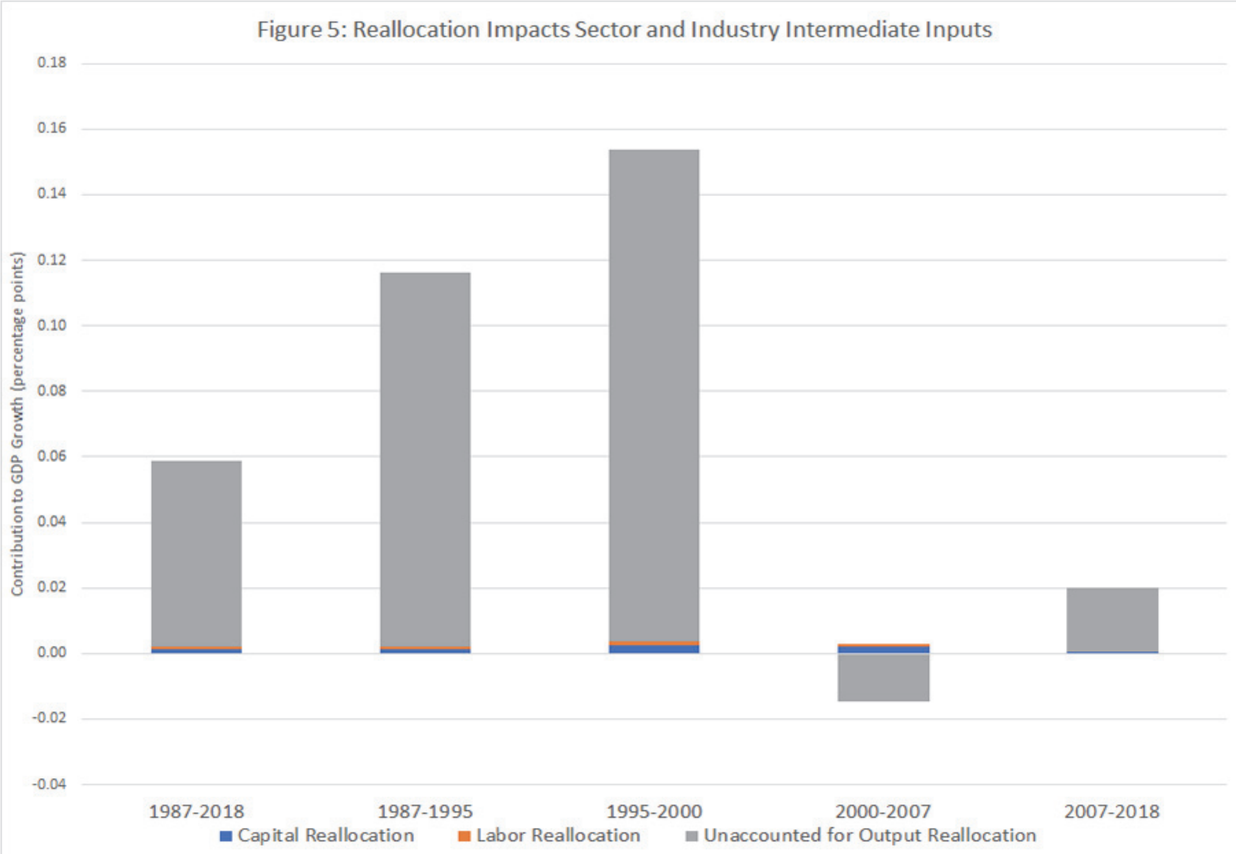
The next level of restrictions is on the different sectors to capture the reallocations across sectors. We required all sectors to have the same price of value added, giving GDP as V_S^{APF} in equation (22) ; this is interpreted as GDP given by an aggregate production function (APF) over aggregate capital and aggregate labor, equivalent to a linear sum of real industry value added. The difference between V_{AS}^{SS} and V_S^{APF} gives us the second set of reallocations in equation (24).

The final component of the reallocation decomposition that we consider is a bit more technical in nature but it is necessary to present a complete accounting of aggregate GDP growth with underlying sectors and industries. This last set of restrictions are due to a consideration of the two ways one can define GDP as a linear sum of value added within our system of tiers and industries. In equation (22) we defined GDP as a linear sum of sector value added requiring them to have a common price, $P_s^V = P_V^{APF}$. However, one could also define GDP as a linear sum of industry value added that we argue would be *even more* restrictive. These two definitions are not identical, that is,

$$V_I^{APF} = \sum_j V_j \neq V_S^{APF} = \sum_S V_s^{PF} \tag{25}$$

These are not equal because real value added at the industry and sector levels (V_j and V_s^{PF}) are both defined by double deflation (equations (5) and (18)).

From equation (8) we see that aggregate real output of a sector equals a simple sum of real output in each component industry. But real intermediate input at the sector level (X_s) does not equal the summation of X_j across industries because each industry within a sector has a different production structure and thus has different weights for each commodity used as intermediate input. Basically, the industry-based aggregation (compared to sector aggregations) allows for more intermediate input reallocations while constraining output reallocations to be zero. Thus, we order V_I^{APF} to be more restrictive than V_S^{APF} and interpret the difference between the two as reallocation of intermediate inputs across industries within sectors. Note that capital and labor reallocations between these two methods are zero because the contributions of capital and labor to GDP growth under both versions of the APF are the same.

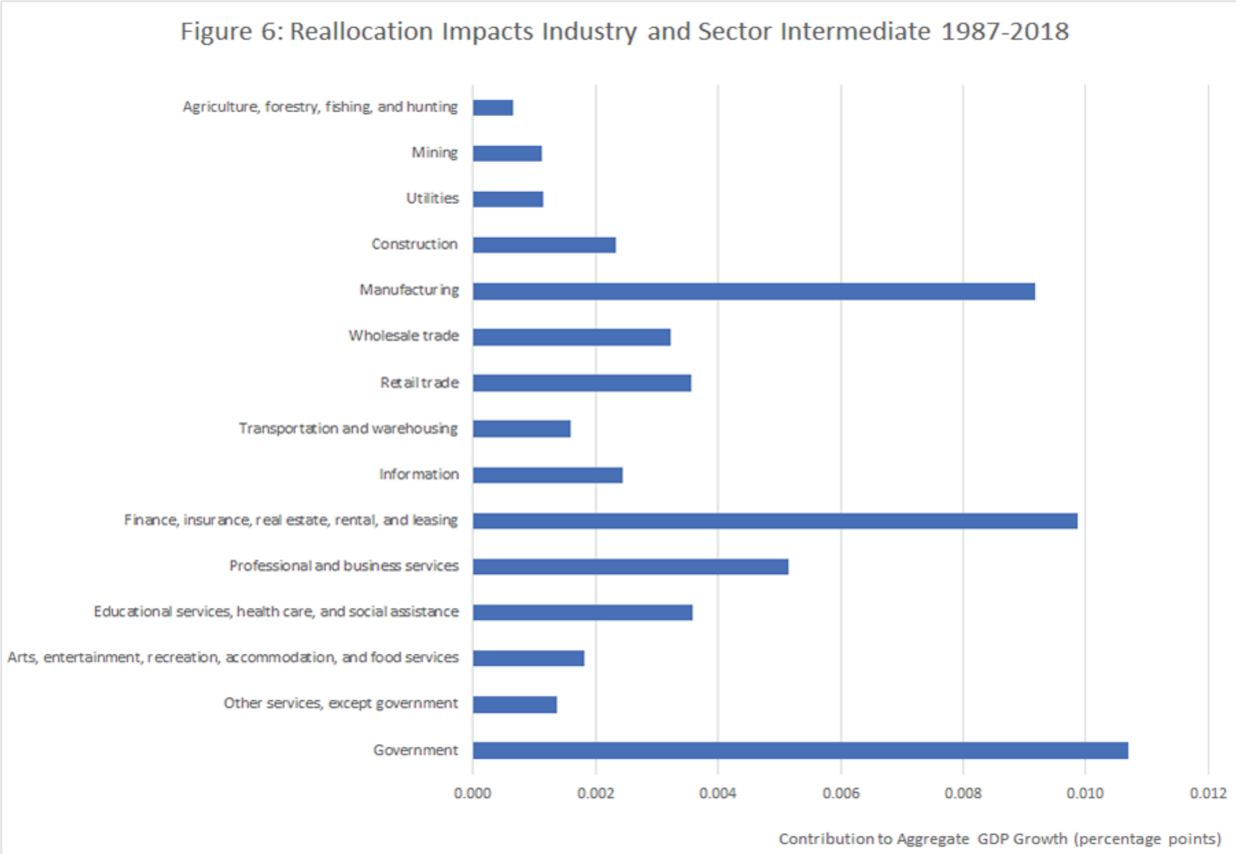


We compute the difference of these two GDPs to capture this last reallocation effect:

$$\begin{aligned}
&\Delta \ln V_S^{APF} - \Delta \ln V_I^{APF} \\
&\approx \sum_s \overline{w_s} \Delta \ln V_s^{APF} - \sum_i \overline{w_i} \Delta \ln V_i^{APF} \qquad : \text{REALVA_APFSS_APF} \\
&= \sum_s \overline{w_s} \frac{\overline{w}_{K,s}}{\overline{w}_{V,s}} \Delta \ln K_s^{APF} - \sum_i \overline{w_i} \frac{\overline{w}_{K,i}}{\overline{w}_{V,i}} \Delta \ln K_i^{APF} (=0) : \text{REALK_APFSS_APF} \\
&+ \sum_s \overline{w_s} \frac{\overline{w}_{L,s}}{\overline{w}_{V,s}} \Delta \ln L_s^{APF} - \sum_i \overline{w_i} \frac{\overline{w}_{L,s}}{\overline{w}_{V,s}} \Delta \ln L_i^{APF} (=0) : \text{REALL_APFSS_APF} \\
&+ \sum_s \overline{w_s} \frac{1}{\overline{w}_{V,s}} \Delta \ln T_s^{APF} - \sum_i \overline{w_i} \frac{1}{\overline{w}_{V,i}} \Delta \ln T_i^{APF} \qquad : \text{REALT__APFSS_APF}
\end{aligned} \tag{26}$$

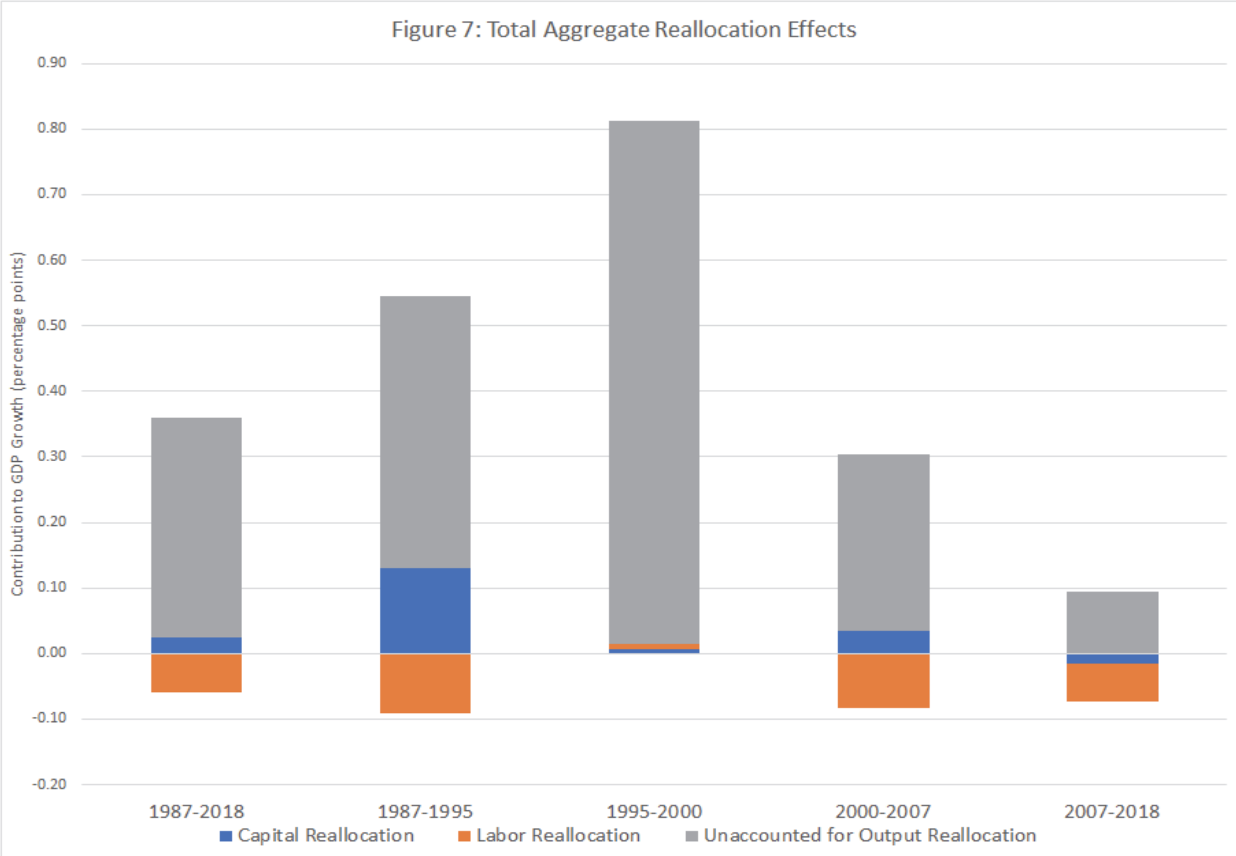
Results

The three components of equation (26) are graphed in figure 5 and show that the reallocation effects from intermediate inputs moving within sectors were relatively small, only 0.06 for the period as a whole. The effects, however, were larger for the early part of the sample, 1987–2000, but negligible post-2000.



Again, the economic interpretation of this reallocation effect is that it captures intermediate inputs moving to industries where they are more productive, holding output reallocation across sectors to be zero. To take a specific example, suppose within the manufacturing sector there were only two industries, computers and paper, and intermediate inputs used in the computers sector were relatively more productive. $\Delta \ln V_S^{APF}$ constrains reallocations across the impact of intermediate inputs used by the computer and paper industries to be zero. $\Delta \ln V_I^{APF}$, on the other hand, embeds the larger contributions of the relatively more productive intermediate in the computers industry. But intermediates are a subtraction from value-added growth, thus VA growth in this case would be slower in the version of the manufacturing sector that is measured using the industry specific intermediates, $\Delta \ln V_I^{APF}$. Thus, the difference between the two measures captures intermediate inputs moving within the manufacturing sector that *ceteris paribus* subtract from GDP growth. Hence, the $\Delta \ln V_S^{APF}$ specification is less restrictive than the $\Delta \ln V_I^{APF}$ model because it allows aggregate VA to be produced without compensating the industry specific intermediates at their marginal productivities. A positive intermediate input reallocation with this ordering indicates intermediate inputs moving to more productive industries within a sector.

Figure 6 gives the reallocations by sector and shows that these intermediate input reallocation effects were positive for all sectors and concentrated within manufacturing; finance, insurance, and real estate; and government.

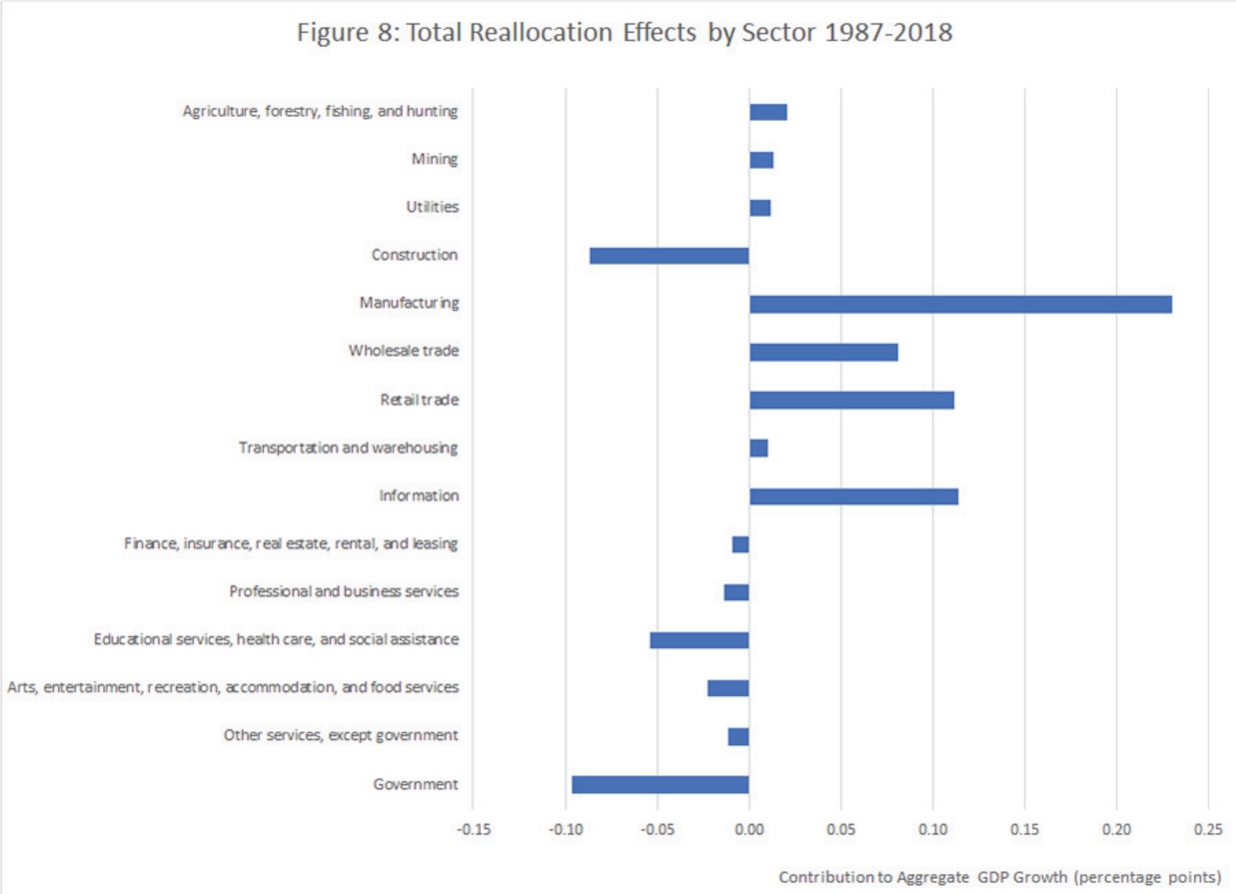


II.5 Total reallocation effects from sectors to industries

In this section we present evidence on the aggregate impact of reallocation effects by comparing the aggregation model that embeds all industry reallocations with the model that is most limiting in terms of the impact of reallocation on aggregate GDP. This corresponds to comparing the sources of growth under the industry model to the sources of growth under the industry version of the APF and amounts to summing up the within-industry, across-sector, and intermediate input effects already discussed. That is, compare $(\Delta \ln V_{AI} - \Delta \ln V_{APF})$ by as the sum of the three components that we noted in the sections above:

$$(\Delta \ln V_{AI} - \Delta \ln V_{APF}) = (\Delta \ln V_{AI} - \Delta \ln V_{ASss}) + (\Delta \ln V_{ASss} - \Delta \ln V_{APFSS}) + (\Delta \ln V_{APFSS} - \Delta \ln V_{APF}) \quad (27)$$

Figure 7 shows that total reallocation accounted for a significant share of aggregate GDP growth over the period 1987–2018 (0.30 percentage point out of 2.39 percent per year). Capital and labor reallocation of all three components accounted for only a small portion of this; thus, the majority of this is explained by output and intermediate input moving to relative more productive sectors. Figure 8 shows that the positive effects were led by the manufacturing, trade, and information sectors. Resources moving to the production of construction; education, health care, and social services; and the government sectors over this time had a significant dampening effect on GDP growth in comparison.



III. Consistent Tiered Accounts

BEA and BLS jointly publish a set of industry accounts covering 63 industries—the Integrated Industry-Level Production Accounts (see Garner et al. 2021). These accounts give the components of equation (1) for each industry, including capital and labor input measures derived from detailed asset classes and demographic data. A separate set of tables are provided for 15 sectors giving value-added data but not TFP estimates. With the framework above one can see that sector accounts for capital, labor, intermediate input, value added and TFP may be constructed to allow a more easily digestible view of the U.S. economy. Our suggestion on how to do this is described in the appendix—it involves indexes that have zero reallocation terms.

IV. Conclusions

The main advantage of our approach of using index number aggregation methods to measure the contribution of reallocations to economic growth is that it avoids unnecessary assumptions that change measured economic history. By comparing empirical estimates of economic growth under the assumption that reallocating resources has no impact on real economic growth to estimates where reallocations of outputs and inputs do impact growth, we see that reallocations have played an important part in U.S. growth since 1987. Reallocations contributed 0.30 percentage point per year on average during the period, out of total GDP growth of 2.39 percent per year. Almost all of these reallocation effects are due to reallocations of output production within manufacturing (for example, to the computer producing sector from other manufacturing sectors) and across sectors to the information and trade sectors; a much smaller portion is due to capital and labor reallocations. These output reallocations were not driven by input reallocations. In our approach, reallocations manifest if output prices differ significantly across industries or input prices (cross classified by detailed type of worker or type of capital) differ across industries. Because our model assumes that prices correspond to marginal products, our results suggest limited barriers to input movements across sectors over this period.

Future research could address the contribution of reallocations further back in U.S. economic history and attempt to address the measurement of reallocations when there is deviation from marginal product pricing, though this most likely would rely on the U.S. statistical agencies building a longer time series of official Industry-Level Production Account data and strong assumptions about how prices differ from marginal products.

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Appendix

In this appendix, we address a practical issue relevant for creating sector-level aggregates. In the main paper, we intentionally chose aggregation schemes based on their aggregation properties to rule in or rule out an economic impact of reallocation. A related issue is how to present sector-level accounts that are consistent with industry-level data and are straightforward to understand from a data user's perspective. For example, a user of the detailed 63-industry data published by BEA may want to know what TFP growth is in the aggregated manufacturing sector without having to consider the complicated relationship between industries within the manufacturing sector and reallocations. In this section, we show that with Tornqvist aggregation of industries to sectors, reallocations are approximately zero and therefore can be seamlessly integrated into the existing BEA industry accounts. With this approach, the underlying economic assumption is that outputs and inputs within sectors can move flexibly across industries. Thus, the economic interpretation of sector accounts constructed using this method is that sector outputs and inputs embed the impact of reallocation across industries in addition to the impact of changes of within-industry outputs and inputs.

We now show that the aggregation equations yield sector accounts with approximately zero reallocations and thus present a near seamless aggregation from industries to sectors, and then from sectors to GDP. Specifically, sector (s) aggregates of outputs and inputs are constructed as Tornqvist indexes over component industries ($i \in s$):

$$
\begin{aligned}
\Delta \ln Q_i &= \overline{w_{Ki}} \Delta \ln K_i + \overline{w_{Li}} \Delta \ln L_i + \overline{w_{Xi}} \Delta \ln X_i + \Delta \ln T_i \\
\Delta \ln Q_i &= \overline{w_{Vi}} \Delta \ln V_i + \overline{w_{Xi}} \Delta \ln X_i \\
\Delta \ln Q_{st} &= \sum_{i \in s} \overline{w_{iQst}} \Delta \ln Q_{it} \\
\Delta \ln X_{st} &= \sum_{i \in s} \overline{w_{iXst}} \Delta \ln X_{it} \\
\Delta \ln K_{st} &= \sum_{i \in s} \overline{w_{iKst}} \Delta \ln K_{it} \\
\Delta \ln L_{st} &= \sum_{i \in s} \overline{w_{iLst}} \Delta \ln L_{it} \\
\Delta \ln Q_{st} &= \overline{w_{Kst}} \Delta \ln K_{st} + \overline{w_{Lst}} \Delta \ln L_{st} + \overline{w_{Xst}} \Delta \ln X_{st} + \Delta \ln T_{st} \\
\Delta \ln V_{st} &= \frac{1}{\overline{\overline{w_{Vst}}}} (\Delta \ln Q_{st} - \overline{w_{Xst}} \Delta \ln X_{st}) \\
\Delta \ln V_{AST} &= \sum_{s} \overline{w_s} \Delta \ln V_{sT}
\end{aligned} \tag{28}
$$

With this specification, reallocations are approximately zero. They are not exactly zero because the shares used in the Tornqvist index are two-period average shares in discrete time. In continuous time, these reallocations would be exactly zero, which is what we show below. To show this, we demonstrate that with contemporaneous value shares, TFP growth at the sector level under Tornqvist weighting equals the gross output weighted-sum of industry TFP growth rates:

$$\begin{aligned}
\Delta \ln T_{st} &= \Delta \ln Q_{st} - [\overline{w_{Kst}}\Delta \ln K_{st} + \overline{w_{Lst}}\Delta \ln L_{st} + \overline{w_{Xst}}\Delta \ln X_{st}] \\
&= \sum_{i\in s} \overline{w_{iQs}}\Delta \ln Q_i - \overline{w_{Kst}}\sum_{i\in s}\overline{w_{iKs}}\Delta \ln K_i - \ldots \\
&= \sum_{i\in s} \overline{w_{iQs}}[\overline{w_{Ki}}\Delta \ln K_i + \overline{w_{Li}}\Delta \ln L_i + \overline{w_{Xi}}\Delta \ln Q_{Xi} + \Delta \ln T_i] \quad - \overline{w_{Kst}}\sum_{i\in s}\overline{w_{iKs}}\Delta \ln K_i - \ldots \\
&= \sum_{i\in s} \frac{P_i^Q Q_i}{P_s^Q Q_s}[\frac{P_i^K K_i}{P_i^Q Q_i}\Delta \ln K_i + .. + \Delta \ln T_i] \quad - \frac{P_s^K K_s}{P_s^Q Q_s}\sum_{i\in s}\frac{P_i^K K_i}{P_s^K K_s}\Delta \ln K_i - \ldots \\
&= \sum_{i\in s} \frac{P_i^K K_i}{P_s^Q Q_s}\Delta \ln K_i + .. + \frac{P_i^Q Q_i}{P_s^Q Q_s}\Delta \ln T_i] \quad - \sum_{i\in s}\frac{P_i^K K_i}{P_s^Q Q_s}\Delta \ln K_i - \ldots \\
&= \sum_{i\in s} \frac{P_i^Q Q_i}{P_s^Q Q_s}\Delta \ln T_i
\end{aligned} \tag{29}$$

We like to emphasize that the definitions of sector measures in (28) are different from the measures in (8–16) that are based on input-output tables of dimension S—tables that impose a common price of intermediate, capital, and labor inputs to all component industries. The indexes in (28) involve the terms w_{iXst} , w_{iKst} and w_{iLst} which contain industry-specific prices.