Imputing Rents to Owner-Occupied Housing by Directly Modelling Their Distribution*

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Abstract

The Bureau of Economic Analysis (BEA) makes the official estimates of the National Income and Product Accounts (NIPAs). Two key aggregates in these accounts are the nation’s gross domestic product (GDP) and the personal income of households. The rental value of owner-occupied housing is an important component of both. It accounts for about 8 percent of GDP and largely determines the rental income of persons. Because this income is a net measure obtained from subtracting housing expenses from the gross rental value of owner-occupied housing services, i.e., space rent, it was only about 3 percent of personal income in 2013. The space rent on owner-occupied housing is considered to be “imputed” because it is not directly measured but is inferred on the basis of its assumed relationship to variables that can be directly measured. BEA’s existing methodology is based on data from a survey that has been discontinued. This paper develops an alternative methodology that can be used to make this imputation by directly modelling the distribution of imputed rents from existing data.

JEL Codes: C82, E01, R31

Keywords: national income and product accounts, imputation, housing, rent

* The views expressed in this paper are solely those of the author and not necessarily those of the U.S. Department of Commerce or the Bureau of Economic Analysis.

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I. Introduction

The Bureau of Economic Analysis (BEA) makes the official estimates of the National Income and Product Accounts (NIPAs). Two key aggregates in these accounts are the nation’s gross domestic product (GDP) and the personal income of households. The rental value of owner-occupied housing is an important component of both. It accounts for about 8 percent of GDP and largely determines the rental income of persons. Because this income is a net measure obtained from subtracting housing expenses from the gross rental value of owner-occupied housing services, i.e., space rent, it was only about 3 percent of personal income in 2013. The space rent on owner-occupied housing is considered to be “imputed” because it is not directly measured but is inferred on the basis of its assumed relationship to variables that can be directly measured. BEA’s existing methodology is based on data from a survey that has been discontinued. This paper develops an alternative methodology that can be used to make this imputation by directly modelling the distribution of imputed rents from existing data.

The paper is organized as follows. Section I presents the history of the housing imputation and summarizes the methodologies used by BEA, other official statistical agencies both in the U.S. and abroad, and private estimators. Section II presents the model-based methodology and describes how it was developed. Section III presents estimates derived from implementing the methodology. Section IV assesses the accuracy of the estimates. Section V discusses alternatives to the model-based methodology. Section VI presents a summary and some concluding remarks. Appendixes describe the details of BEA’s existing methodology, the details of how a gamma distribution was fit to rent data, and reasons for using data from the American Community Survey (ACS).

A. Rationale for the housing imputation

The notion that owner-occupied houses are capital goods that yield valuable services to their owners that should be included in measures of their income has been common since the late 19th century. This consideration alone might cause statisticians who estimate national income to impute values to these services as part of their measures. However, there is another important reason for doing so - the belief that estimates of national income should be invariant to changes in institutional arrangements such as whether houses are owner-occupied or tenant-occupied. The reasoning for this has often been framed in terms of the following example. Suppose that families A and B live next door to each other in identical owner-occupied houses. In the absence of any imputation, the rental services that these families obtain from their houses are not included in GDP. Now suppose that family A moves out of its house and into B’s and pays family B a rent of $X. Family B moves out of its house and into A’s and pays family A a rent of $X. Each family has net rental receipts of zero. They each pay the other $X but receive $X back in return. The rental payment that each family receives from the other is included in GDP so that GDP has increased by $2X. National income accountants believe that nothing was really changed by each family’s moving into its neighbor’s house so that GDP should have been unaffected by this change in which houses they live in. However, this result will only be obtained if the services of owner-occupied and tenant-occupied houses are both valued in the same way.
Ever since the NIPAs were first published in 1947, they have included an imputation for the services of owner-occupied housing. Since then, BEA has always estimated the current-dollar value of housing services as the product of the physical number of housing units and the mean rent per unit. Separate but analogous estimates are made for tenant-occupied and owner-occupied units. The mean rent for tenant-occupied units is directly estimated while the mean rent for owner-occupied units is imputed using more indirect methods. The imputed rent is an “opportunity cost,” the forgone rent that the owner-occupier could have earned had the owner chosen to rent out his house. This “space rent” is included in personal consumption expenditures (PCE) on the product side of the accounts. PCE does not include any costs of owning homes such as the mortgage interest paid, depreciation, and expenditures on maintenance and repair that are of the kind that a landlord would make. These expenses are subtracted from space rent to obtain a (net) rental income of persons that is included in personal income on the income side of the accounts. The housing imputation is also used in the national accounts of most countries. The first United Nation’s System of National Accounts (SNA), which was published in 1951, included this imputation. Similar concepts are also used in the construction of consumer price indexes and international purchasing power parities.

B. Recent NIPA methodology

For the last several decades, BEA has imputed the mean contract rent for owner-occupied housing using the “rent-to-value” approach. This approach assumes that owner-occupied units with similar market values as rental units have similar rent-to-value ratios (Mayerhauser and McBride, 2007). More specifically, BEA assumes that if a renter-occupied unit and an owner-occupied unit have identical market values, then the market rent for the two units will also be identical.

BEA implements the methodology by constructing a single, national rent-to-value schedule that is applied to all owner-occupied housing units. The schedule shows the rent-to-value ratio that is appropriate for units at each housing value. In the past, schedules were estimated each decade using data from the Residential Finance Survey (RFS) that was conducted every ten years by the Census Bureau in conjunction with the Decennial Census of Housing. (The details of the procedure are described in greater detail in Appendix I of this paper.) The RFS was unique in that it was a survey of property owners and their managers and not just households. These owners and managers were able to report both the market value of buildings that they were renting out and the amount of rent that they received. The 2015 Rental Housing Finance Survey has only a very small sample of single-family units and does not contain data on their market value.

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1 The imputation, however, was not included in BEA’s earlier estimates of either national income or national product.
2 The 2008 SNA (paragraph 1.65) cites the opportunity cost principle as the foundation of the accounts. “In the SNA, however, the concept of opportunity cost as defined in economics is employed. In other words, the cost of using or using up, some existing asset or good in one particular process of production is measured by the amount of the benefits that could have been secured by using the asset or good in alternative ways. Opportunity cost is calculated with reference to the opportunities foregone at the time the asset or resource is used, as distinct from the costs incurred at some time in the past to acquire the asset.”
Unfortunately, the funding for these surveys was cut and the last RFS was conducted in 2001. Since then, BEA has been extrapolating its results. This extrapolation can be problematic in the presence of differential housing inflation. To see this, examine chart 1. Rent-to-value schedules generally slope downward and to the right as rent-to-value ratios generally decline with a house’s market value. The typical shape is exhibited by the one based on the 1981 RFS, the blue line in the chart. The schedules for the 1991 RFS and the 2001 RFS are to right of the earlier one. This is only to be expected. Suppose that a house is at both the 50\textsuperscript{th} percentile of rents and the 50\textsuperscript{th} percentile of market values. If the house’s rent and the house’s market value both increase at an annual rate of X percent per year, the house’s rent-to-value ratio will remain unchanged over time but the house’s point on the rent-to-value schedule will move to the right as the house’s market value increases over time.

![Chart 1 - Rent to Value Ratios](image)

It is now more than 15 years since the last RFS was conducted. The true schedule for the current year has most probably not only moved to the right, it has most likely changed shape. That this is, indeed, the case is evidenced by data on paid rents from the ACS. The rate of inflation in the mean rent for single-family houses is often quite different from the comparable rate for the median rent in the same year. In some years the two inflation rates have differed by as much as two percentage points. This indicates that significant changes in the shape of the rent-to-value schedule are occurring. The impacts of such changes cannot be captured using simple extrapolations.

C. Estimates of owner-occupied housing services made by other U.S. statistical agencies

Housing imputations that are similar to BEA’s have been made by other statistical agencies. The first U.S. Census of Housing, which was conducted in 1940, included estimates of the rental value of owner-occupied houses. The estimates were made by individual enumerators sent out to the houses. Surprisingly, the enumerators were not asked to estimate the market value of these houses because it was considered too difficult a task.
Estimates of the quantity and price of owner-occupied housing services are implicitly made by the Bureau of Labor Statistics (BLS) because they are needed to construct the consumer price index (CPI). Poole, Ptacek, and Verbrugge (2005) note that from the early 1950’s until 1983, BLS used the payments or cash flow approach to measure changes in the price of owner-occupied housing services. The services were measured by the sum of the various out-of-pocket expenditures associated with owning a house, such as mortgage payments and required maintenance. As inflation accelerated in the 1970’s, the BLS measure came under severe criticism and BLS sought to replace it. One of the two major alternatives that BLS considered was the “user cost” approach, which is derived from the principle that, in equilibrium, the price of a capital asset is equal to the discounted sum of the values of the future services it will yield. Depreciation and an imputed return to capital are the two most important cost elements in this approach. Others include expenditures on maintenance, property taxes, and other operating expenses as noted by Gillingham (2003) and Katz (2009). An experimental user cost measure was published by BLS. However, in the late 1970’s and early 1980’s the inflation adjusted or “real” interest rate was tremendously volatile. This caused the user cost measure to become inoperable and all proponents dropped their support for it. Verbrugge and Garner (2009a) indicate that this type of problem still plagues empirical estimates of housing services based on the user cost method.

At this point, the only viable measure for the BLS was the one based on the rental equivalence approach. According to Poole, Ptacek, and Verbrugge (2005, p. 9), “The rental equivalence estimate of the costs of shelter for a homeowner is the amount that the homeowner would have to pay in order to rent their home (or a comparable home), or as the BLS Handbook of Methods puts it: ‘In essence, [owners’ equivalent rent] measures the change in the amount a homeowner would pay to rent, or would earn from renting, his or her home in a competitive market. It is a measure of the change in the price of the shelter provided by owner-occupied housing.’”

Note that in the construction of the CPI, BLS does not actually estimate the market rents of comparable properties. It estimates the percentage change in the rent of comparable properties. Consequently, BLS does not regularly publish a time series that can be directly compared with BEA’s estimates of space rent. Passero, Garner, and McCully (2015, p. 193) note that in order to facilitate research into the difference between BEA and BLS estimates of consumer expenditures, BLS has constructed a special tabulation of their estimates that includes an estimate of the aggregate space rent on owner-occupied dwelling units.

D. Official estimates made outside the U.S.

Outside of the U.S., particularly in Western Europe, the rental value of owner-occupied housing is estimated using the “stratification method.” According to the Eurostat-OECD Methodological Manual on Purchasing Power Parities (Luxembourg: Publications Office of the European Union, 2012),

“The preferred method of determining rental equivalents is the stratification method whereby the housing stock is broken down by type, size, quality and location into strata and combined with information on actual rents paid in each stratum. More precisely, the number of owner-occupied dwellings in the stratum is multiplied by the average rent
paid for rented accommodation in the stratum to arrive at the total imputed rent for the stratum. Summing the imputed rents across strata gives the total imputed rent for the country.”

“Implementation of the stratification method requires the existence of a well-organised market for rented housing. In the absence of such a market, the second best method, the user cost method, has to be employed. This entails estimating imputed rents by summing all the costs that owner-occupiers incur in owning their dwellings: intermediate consumption, compensation of employees, consumption of fixed capital, net operating surplus and other taxes (less subsidies) on production.” P.138

In Eastern Europe, housing markets are generally not well-organized so that the stratification method could not be used when these (candidate) countries acceded to the European Union. Consequently, Eurostat formed The “Task force on estimation methods for dwelling services in the Candidate Countries” to define a user cost approach and to consider practical options for its implementation by these countries. Katz (2009) noted that one of the most important recommendations made by the task force was that real rates of return on both dwellings and land should be assumed to be 2.5 percent. The use of these constant real rates eliminated the problems caused by volatile and negative real rates. They also caused the estimated share of GDP going to housing services in the various candidate countries to be roughly the same as the comparable shares in the countries of Western Europe. Nevertheless, it is difficult to justify the use of fixed interest rates with the standard theoretical apparatus.

E. Unofficial estimates

Davis, Lehnert, and Martin (2008) present estimates of the rent-to-price (i.e., rent-to-value) ratio that are comparable to BEA’s. This study is notable for its use of national data. The authors use Census micro-data on rents paid to estimate a hedonic function that relates rents to the characteristics of the rental units. They then used the estimated equation and data on comparable characteristics of owner-occupied houses to estimate rent-to-price ratios for those houses. The results are quite different from BEA’s. Their estimated national rent-to-price ratio was 4.6 percent in 2000. It was used to impute a mean contract rent for owner-occupied housing in 2000 that is 32 percent less than BEA’s. Several studies examined data for a small number of geographic areas in the U.S. One of the more notable of these is by Heston and Nakamura (2009a), who found rent-to-price ratios that were similar to BEA’s.

II. Overview of the model-based methodology

In the model-based methodology developed here, data on the distribution of rents actually paid on single-family houses that are rented are used to construct a distribution of imputed rents for owner-occupied houses. The method is implemented using data from the Census Bureau’s largest sample surveys based on the belief that this will lead to the most accurate estimates. As much as possible, data

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3 A time series of imputed rents based on the methodology from this article is published on the website of the Lincoln Institute of Land Policy.
is taken from surveys that are conducted annually. This reduces the need for extrapolations over long periods of years, which are inherently inaccurate.

The key methodological assumption is the logical extension of the one that underlies BEA’s existing estimates. BEA currently assumes that if two houses have similar market values, the market rents for the two houses will also be similar. We make the relationships stricter by substituting the word “identical” for “similar.” We also make the additional assumption that if house B has a higher market value than house A, then its market rent will also be higher. This guaranties that a house that is at the \( i^{th} \) percentile of the distribution of market values will also be at the \( i^{th} \) percentile of the distribution of market rents.4

**A. The distributions of market values of tenant- and owner occupied houses in 2001**

Data on the correspondence between the market values of single-family houses that are rented out and those that are owner-occupied is available from the 2001 RFS. Table 2.1 of the 2001 RFS presents data on the distribution of market values of owner-occupied houses while table 4.1 presents comparable data on the distribution of market values for single-family tenant-occupied houses. The distributions of these market values are shown in chart 2.

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4 Suppose this additional assumption was not made. We could then have a situation where houses A, B, and C, which respectively have market values of $295,000, $300,000, and $305,000 are found to respectively have rents of $1,600, $1,500, and $1,600. If house D, which is owner-occupied, was found to have a market value of $300,000 we would be insisting that its market rental value must be $1,500 and not $1,600. The logic behind such an insistence is obscure at best.
From this chart we can see that houses with a market value of $125,000 are at about the 50th percentile of the distribution for owner-occupied houses and at about the 71.5 percentile of the distribution for tenant-occupied houses.

The information underlying the two lines is combined to obtain the blue line on chart 3. Each point on that line is for a different market value. For a given market value, the associated point on the line shows the percentile of the tenant-occupied distribution for houses with that value and the corresponding percentile of the owner-occupied distribution for houses with that value. The red line shows those points where the percentiles of the two distributions are equal to each other. If the distributions of the market values of owner-occupied and tenant-occupied were exactly equal to each other, then the blue line would be where the red line is. Comparable data from the 1991 RFS is used to construct the green dashed line shown on the chart. The curves for the 1991 and 2001 distributions are remarkably similar, especially in their middles. This suggests that the relative relationships shown on this chart may be stable over time. The methodology developed later in this section is based on the assumption that this stability will continue in the future.

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5 A curve based on the 1981 RFS could not be constructed because that survey lacked data on the market value of single-family rented homes.
B. The distribution of owner-occupied rents in 2011

The results of the preceding section are used to estimate the distribution of owner-occupied rents in 2011. Micro-data from the 2011 ACS is used to measure the distribution of actual contract rents paid on single-family houses that have cash rents. (Mobile homes are excluded from this tabulation.) Specifically, we estimate the rent at each of the percentiles of tenant-occupied rents using the data from the 2001 RFS. We assume that, relative to each other, the distributions of market values of owner-occupied houses and tenant-occupied houses are the same as in 2001, i.e., the distribution given by the blue curve in chart 3 is assumed to also hold in 2011. Because the relative percentiles for rents are assumed to be the same as those for market values, we know the percentiles of the owner-occupied distribution of rents for 2011 that corresponds to each of the tenant-occupied percentiles. We then interpolate between these points to obtain the percentiles for all contract rents in increments of $100. This gives us the “actual” distribution of owner-occupied rents shown in chart 4.

C. The statistical model

The next step was to develop a statistical model of the distribution of owner-occupied contract rents. The “actual” distribution has a few salient characteristics. Most of the distribution is clustered just below and above the median value of $1,068. The left hand tail near the zero rent value is short and thick. The right hand tail, where the high rents are, is rather long and thin. These characteristics are embodied in the gamma distribution. This distribution is used in estimating waiting times, the amount of insurance claims, and the distribution of income. One aspect in its favor is that its right hand tail is smaller than the tails of similar distributions like the log-normal.
The gamma distribution is a function of two parameters, $\alpha$ and $\beta$. Mathematically, it is given by:

1. **Probability density**
   \[
   f(x, \alpha, \beta) = \frac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} e^{-\frac{x}{\beta}}
   \]

2. **Lower cumulative distribution**
   \[
   P(x, \alpha, \beta) = \int_0^x f(t, \alpha, \beta) \, dt
   \]

3. **Upper cumulative distribution**
   \[
   Q(x, \alpha, \beta) = \int_x^\infty f(t, \alpha, \beta) \, dt
   \]

Alpha ($\alpha$) is a shape parameter and beta ($\beta$) is a scale parameter. Because it is merely a scale parameter, without loss of generality, we set $\beta$ equal to 1. For a gamma distribution with given values of $\alpha$ and $\beta$, it is straightforward to estimate the cumulative density of the distribution for each of any desired number of equally spaced points on the distribution.\(^6\)

For any given shape, i.e., for any given value of alpha, i.e., the distribution can be fit given knowledge of two points on it. That is, the estimated distribution is forced to be equal to the actual distribution at two points. The two points that appear to be used most often are the origin and the median (50th percentile). However, there is little to be gained by forcing the estimated distribution to go through the origin. Doing so will cause the estimated distribution to be very accurate at extremely low rents. However, this will do little to improve the accuracy of the estimated mean rent.

We choose not to force the distribution through the origin so that the 0 percentile is estimated to occur at a rent that is greater than zero. Instead, the actual and estimated distributions are forced to have the same values at the 50th and 90th percentiles of the distribution of actual imputed rents. This causes the estimated distribution to better fit the actual distribution near the right-hand tail, where small differences can have a large impact on the estimated mean rent.

Another way of describing this methodology is that to fit the desired distribution of rents, we need to know two factors. The first is the rent at the first point on the distribution, i.e., the smallest non-zero rent. The second is the difference between the rents at successive points on the distribution. The values of these two factors can be determined if we know the rents at any two points on the distribution. These points were chosen to be the values at the 50th and 90th percentiles of the distribution of owner-occupied rents. By interpolating the values underlying the blue line in chart 3, these values are found to occur, respectively, at the 71.4 and 94.2 percentiles of the distribution of tenant-occupied rents. From the data used to estimate contract rents paid in 2011, these rents are found to be, respectively, $1,072 and $1,908. The methodology that is given in Appendix II of this paper is then followed to calculate the values of the two factors needed to fit the distribution for a given value of alpha.

\(^6\) Programs are available over the internet for these calculations. The calculations for this paper were made using one that is available at: http://keisan.casio.com/exec/system/1180573216. The formula given above is taken from its write-up.
D. Adjusting the upper tail of the distribution

Estimates at the high end of the distribution of imputed rents are problematical. According to the 2011 ACS, there are 3.0 million owner-occupied houses with market values of at least $750,000. A straight-forward use of BEA’s existing methodology based on the 2001 RFS would lead to the conclusion that all these units have a rent-to-value ratio of 6.29 percent. They would all then be imputed to have contract rents of over $3,930 per month. An analysis of the detailed data and top-coding procedures used for the ACS reveals that in 2011 of the more than 12 million single-family houses that were rented out, less than 50,000 of them received rents that are at or above this value. This presents us with a thorny conceptual question. What is the mean market rental value for the 3 million high-end houses? How few units must actually be paying rents of more than $3,930 per month before we believe that this is an overestimate of the true market value? Is it 1,000,000 units, 100,000 units, or 10,000 units? There is no good answer to this question.

It is not clear why so few tenant-occupied single-family houses are reported to have high rents.\(^7\) Given the uncertainty about what the true situation is, no adjustments were made for under reporting.

This paper handles the problem of high-end rents by making a supplemental adjustment. About 2 percent of single-family houses that are rented out have top-coded rents in the 2011 ACS. Each state has its own top codes. Top-coded units have their actual rent replaced by the mean top-coded value for their state. In this paper, the top-coding problem is ignored for the bottom 99.8 percent of the distribution of paid rents and top-coded values are used as if they were real estimates. Units in the upper 0.2 percent of the distribution are treated as having double the mean value of the state with the highest top-code. In other words, these houses are assumed to have a mean rent of $7,600 per month. This adjustment is probably larger than what is strictly warranted by the survey data.\(^8\)

E. Fitting the distribution

The “actual” distribution of owner-occupied contract rents for 2011 was fit using the gamma distribution and five different values of alpha: 1.9, 2.5, 3.0, 3.5, and 4.0. In table 1, the means of the fitted distributions are compared with the actual one. The distributions with alphas of 3.5 and 4.0 have by far the best fits. Their means differ from the actual mean of $1,150 by only about 2.0 percent and about 1/3 of these differences are due to the special adjustment that is made in the upper tail.

<table>
<thead>
<tr>
<th>Alpha</th>
<th>1.9</th>
<th>2.5</th>
<th>3.0</th>
<th>3.5</th>
<th>4.0</th>
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<tbody>
<tr>
<td>Percent difference from actual mean</td>
<td>4.54</td>
<td>6.36</td>
<td>2.75</td>
<td>2.20</td>
<td>1.74</td>
</tr>
</tbody>
</table>

\(^7\) Curiously, the Census Bureau has no problem finding large numbers of respondents who indicate that their houses are worth over $2 million.

\(^8\) For each state, the ACS microdata reports the number of units that are top-coded, the mean rent for top-coded units, and the minimum rent for units to be top coded. A detailed analysis of this data for 2011 suggests that no more than about 0.2 percent of all single-family houses had paid rents in excess of $3,400 per month.
These two distributions are shown along with the actual one in chart 5. There is little to choose between the two fitted distributions. The one based on an alpha of 3.5 has a higher minimum rent (the rent at the zero percentile), a higher maximum rent (at the 99.8 percentile), and a higher spike in frequencies near the median value. For rents just higher than the median until the upper tail, the estimated frequencies are nearly identical. Although it has a slightly higher overall difference from the actual (‘estimated’) mean, the distribution with an alpha of 3.5 is used in all subsequent calculations because it seems to yield a better fit near the median and has higher values in the extreme upper tail.9

III. Resulting estimates

The series of calculations described above were repeated for all years between 2004 and 2014 using data from the ACS. They were also repeated for 1990 and 2000 using data from the Decennial Census of Housing (COH).10 All estimates were made using an alpha of 3.5. The results are shown in table 2 below.

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9 As alpha is increased, the area in the left-hand tail increases while the area in the right-hand tail decreases. This goes against our priors especially because we are already making an adjustment at the right-hand tail for a supposed undercount. Extremely low rents are also implausible. These considerations caused us to conclude that the apparent gain in accuracy by using a higher value of alpha is illusory.

10 For these two surveys, the assumptions regarding the upper tail were modified to reflect the lower general value of rents. For 1990, the upper 0.2 percent of single-family units are assumed to have a mean rent of $4,023; for 2000, units in the upper 0.2 percent are assumed to have a mean rent of $5,000.
Table 2 - Effects of Model-Based Measure on Space Rent and Rental Income of Persons (in billions of dollars)

<table>
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</thead>
<tbody>
<tr>
<td>BEA estimate of space rent</td>
<td>395.2</td>
<td>740.8</td>
<td>934.3</td>
<td>1010.6</td>
<td>1077.7</td>
<td>1113.8</td>
<td>1158.6</td>
<td>1183.2</td>
<td>1189</td>
<td>1208.2</td>
<td>1230.1</td>
<td>1278.6</td>
<td>1325.6</td>
</tr>
<tr>
<td>BEA estimate of the rental income of persons</td>
<td>-6.8</td>
<td>111.9</td>
<td>160.5</td>
<td>139.1</td>
<td>108.7</td>
<td>82.9</td>
<td>134.3</td>
<td>202.5</td>
<td>253.3</td>
<td>310.8</td>
<td>340</td>
<td>378.3</td>
<td>417.4</td>
</tr>
<tr>
<td>Model-based estimate of space rent</td>
<td>377.5</td>
<td>605.9</td>
<td>746.8</td>
<td>801.9</td>
<td>854.3</td>
<td>922.0</td>
<td>967.4</td>
<td>981.9</td>
<td>1002.1</td>
<td>1029.5</td>
<td>1051.5</td>
<td>1083.5</td>
<td>1109.2</td>
</tr>
<tr>
<td>Model-based estimate of the rental income of persons</td>
<td>-24.5</td>
<td>-23.1</td>
<td>-27.0</td>
<td>-69.6</td>
<td>-114.7</td>
<td>-108.9</td>
<td>-56.9</td>
<td>1.2</td>
<td>66.4</td>
<td>132.2</td>
<td>152.5</td>
<td>183.2</td>
<td>201.0</td>
</tr>
<tr>
<td>Difference between BEA and model-based estimates of space rent (and rental income)</td>
<td>-17.7</td>
<td>-135.0</td>
<td>-187.5</td>
<td>-208.7</td>
<td>-223.4</td>
<td>-191.8</td>
<td>-191.2</td>
<td>-201.3</td>
<td>-186.9</td>
<td>-178.6</td>
<td>-187.5</td>
<td>-195.1</td>
<td>-216.4</td>
</tr>
</tbody>
</table>

The estimates of space rent obtained with the model-based methodology are generally quite different from BEA’s existing estimates. However, the model-based estimate for 1990 is surprisingly close to BEA’s; it is only $18 billion lower. The estimate for 2000 is $135 billion lower. Although the difference in the levels of the two series generally increases thereafter, the percentage difference between them does not vary much.

IV. On the accuracy of the model-based estimates

A. Imputed rents verses overall paid rents

One test of the reasonableness of the model-based estimates is to compare their mean values with BEA’s estimates of the mean values of rents paid on all tenant-occupied dwellings. As shown in chart 6, the model-based estimates are about 50 percent higher than the mean paid rent. These percentages are lower than the comparable ones obtained using BEA’s imputed rents for 1990 through 2014. However, they are higher than the percentages obtained using BEA’s imputed rents for years prior to 1980.

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11 The BEA estimates shown in this paper are consistent with BEA’s 2015 annual revision and do not reflect changes introduced after June 2016. The estimates shown in table 2 are solely for owner-occupied housing.
B. Rents paid on single-family houses versus overall paid rents

The ratio of the mean imputed rent to the mean paid rent for all dwelling units (denoted by A) can be expressed as the product of two ratios: the ratio of the mean paid rent on single-family houses to the mean paid rent on all dwelling units (denoted by B) and the ratio of the mean imputed rent to the mean rent paid on single-family houses (denoted by C). In order to obtain any given value of A, the lower the value of B, the larger the value of C must be. To judge what are reasonable values for A, we examine the available evidence on what the values of B and C are.

The Census Bureau has extensive data related to the historical values of the first ratio, B. The data clearly shows that this ratio is barely larger than one, i.e., the mean rent paid on single-family houses is not much higher than the mean rent paid on all dwelling units. Chart 7 shows four decades of data obtained from the AHS on the median gross rent of single-family detached houses as a percent of the median gross rent of all tenant-occupied units.\(^\text{12}\) Because gross rents for single-family detached houses have increased rapidly relative to the rents for all dwelling types since 1995, it might appear that by 2011 they were commanding substantially higher rents than the overall mean rent.\(^\text{13}\) Such an interpretation would not be correct. The data examined above is for gross rents while BEA measures its

\(^{12}\) Prior to 1985 the AHS was called the Annual Housing Survey. After then, the survey was conducted only every other year and it was renamed the American Housing Survey.

\(^{13}\) The ACS also reports data on gross rents. This data is similar to the AHS data on gross rents. Thus, the 2011 ACS shows that the mean gross rent for detached single-family houses was 14 percent higher than the mean gross rent for all dwelling units.
aggregates using contract rents. The two differ in that gross rents include expenditures on utilities by tenants that are not in the contract rent. When we examine data on contract rents, the picture is substantially different. Chart 8 presents data from the ACS from 2004 through 2014. It shows that the mean contract rent for detached single-family houses was only 5 percent higher than the overall mean in 2011 and only 3 percent higher than it in 2014. The smaller percentage found when contract rents are used results from the fact that utility expenditures are much higher for single-family houses than for units in multi-family dwellings.

C. Rents on larger versus rents on smaller houses

We now address the question of what the value of C is. More specifically, we want to know whether large single-family houses rent for substantially more than smaller single-family houses do. Because C involves an imputed value, we have only indirect information of what its value is. Chart 9 presents data from several years of the AHS that compares median gross rents of detached single-family
houses and mobile homes by size of house. The rents for each size category are shown relative to those of the 1,000 – 1,499 square foot category. The overall median size has always been within this size category. Rents for the largest size categories generally exceed those of the comparison size category by no more than 30 percent. (Because utility expenses are much higher in the larger houses, the percentages would be much smaller if they were made using contract rents instead of gross rents.) Most importantly, these percentages have not increased since 1991. Interestingly, in some years the highest median rents are not found in the largest size category.

Data by size of house is not available in the ACS. However, data by the number of bedrooms is available in it. In chart 10 we see that detached 4-bedroom houses rent for about 25 – 31 percent more than do 3-bedroom houses. These results are very similar to those obtained in the AHS comparisons using square footage.

For the period 2004 through 2014, the mean imputed rents obtained using the model-based methodology ranged from being 130 to 132 percent of the value of the comparable mean paid rent for single-family houses. These values are close to the relative effects on rent of more square footage and more bedrooms found in the comparisons made above using data from the ACS and AHS. This demonstrates that the model-based estimates are consistent with the data in the ACS and AHS. Furthermore, the stability of the ratio of the model-based estimates to the observed mean for paid rents suggests that there has been little change in the shape of the distribution of paid rents on single-family homes from 2004 through 2014.
D. The implied rent-to-value ratio

An additional test of the reasonableness of the model-based estimates is to examine the implied rent-to-value ratios for 2011. These values are constructed by dividing the estimated annual rent for a given percentile of the distribution of owner-occupied rents for 2011 (using the model-based methodology) by the market value for owner-occupied houses at that percentile based on data from the 2011 ACS. As shown on chart 11, at any given market value below the median, the rent-to-value ratio implied by the model-based methodology generally exceeds the ratio obtained using the 2001 RFS. At any given market value above the median, the implied rent-to-value ratio is generally less than obtained using the 2001 RFS. Note, the model-based methodology only implies rent-to-value ratios that are below 4 percent for houses that are in the upper 5 percent of the distribution, i.e., those with values above $646,000.
E. Evidence from other studies

Results published in other studies provide support for the model-based estimates. The 2013 edition of Eurostat’s Handbook on Residential Property Prices Indices (RPPIs), which was coauthored by Erwin Diewert, provides evidence that is consistent with the schedule of rent-to-value ratios implied by the model-based estimates. The handbook cites two studies by Heston and Nakamura, which were published in 2009, as showing that the rent-to-value ratio for Washington, DC, was 6.9 percent for a $200,000 house and 4.3 percent for a $500,000 house. Likewise, it cites Garner and Verbrugge (2009b, 178) as showing that U.S. data for 2004-6 was consistent with rent-to-value ratios that declined to 3.3 percent for a $900,000 house. The rent-to-value ratios that are implied by the model-based estimates are remarkably close to all three of these percentages. Specifically, the implied percentages are 7.0 percent for a $200,000 house, 4.7 percent for a $500,000 house, and 3.5 percent for a $900,000 house.

V. Other alternative methodologies

A. A shortcut methodology

Practitioners who wish to replicate the model-based methodology for other years may find it a bit difficult. Fortunately, we were able to develop a shortcut methodology that produces estimates that nearly replicate those obtained using the unabridged methodology. The shortcut is a simple weighted average of the two rents that were used to fit the gamma distribution. A weight of 7/8 is given to rent at the 71.4 percentile of the distribution of paid rents on single-family houses and a weight of 1/8 is
given to the rent at the 94.2 percentile.\textsuperscript{14} The shortcut methodology replicates the results of the unabridged methodology with surprising precision. It was used to determine the mean imputed rent in thirteen different years and annual growth rates for eight of them. In ten of these years, the shortcut’s estimate was within $\$1$ of the monthly rent obtained using the methodology. Annual growth rates of imputed rent obtained with the shortcut were also amazingly close to those obtained using the unabridged methodology. The shortcut always produced annual growth rates that were within 0.1 percentage point of the annual growth rates obtained with the methodology.

It takes very little time to implement the shortcut. One only needs to determine the values of paid rents at two percentiles of the distribution. The gamma distribution does not have to be fit. Using the shortcut means that one is implicitly using the parameters of the gamma distribution that underlies the unabridged methodology. These parameters were selected using data for 2011 and they produced excellent results for estimates as early as 1990.

\textbf{B. Other alternatives}

One alternative to the model-based methodology is to assume that rents on owner-occupied houses differ from those on tenant-occupied houses solely because they have a different set of characteristics. If we could determine how actual rents vary with these characteristics, we could use that set of relationships to determine what the imputed rents should be. Thus, we could follow Aten, Figueroa, and Troy (2012), who largely relate rents to the number of bedrooms. However, we have seen that the type of building is also an important factor in determining rents. So are the dwelling’s age and whether it has the latest appliances or was recently renovated. This leaves the key question of how can we determine which of these factors are truly important.

To some extent, this problem can be resolved by estimating a hedonic relationship between rents and characteristics. However, such an approach can never be truly satisfactory for housing services. It will always lack the influence of the most important variable – location. Unless you know that a condominium apartment with no bedrooms is located in Manhattan, the hedonic method will lead you to impute a very low rent to the dwelling unit. The model-based methodology is not subject to this problem. Condominiums in Manhattan have very high market values. Once we know that the dwelling unit’s market value is very high, we will impute a very high rent to it.

The methodology based on the hedonic method also suffers from an important conceptual difficulty. It may estimate different rents for dwelling units that have different numbers of bedrooms but which have the same market value. This would be inconsistent with the principle that underlies BEA’s current imputation - that dwelling units with the same market value will rent for the same amount.

\textsuperscript{14} The weights were determined by first finding the weights required for each year’s estimates to precisely match the results of the unabridged methodology. In all years, the implied weights for the lower rent were extremely close to 7/8.
Another alternative to the model-based methodology lies in the use of “big data.” It may be possible to obtain data via the internet on transactions prices (both rent and market values) for small geographic areas. Ideally, we would also like to have data on the characteristics of the houses that are being rented and sold. Specifically, we would need rents for rented single-family houses and market values for owner-occupied houses, broken down by both whether the house was attached or not and by a measure of its size (the number of bedrooms or square feet). Without this data, this method relies on geographic location as being the sole determinant of rents. However, if the stock of houses within the geographic area is heterogeneous, the method runs the risk that actual rents from low-valued houses may be lined up with owner-occupied houses that have a much higher market value. In general, the smaller the geographic area for which data is being collected, the more homogenous will be the housing stock within each area, and the less likely it will be that the above concern will result in large errors. Given the diversity of the housing stock within certain counties, it appears that successful application of this method would need data for very small areas, such as zip codes or census tracts.

There is a potential pitfall that should be avoided when data is collected over the internet. When data on market values of houses is not available, providers of the data will sometimes substitute data on the appraised value of houses. However, assessed values are a poor substitute for transaction prices. Changes in assessed values lag well behind changes in actual market values and the percentage change in assessed values in any given year may have little relationship to the percentage change in market values in that year.

VI. Summary and conclusions

Because of the termination of the RFS, this paper develops a model-based methodology to impute the rental value of owner-occupied houses. A number of housing surveys provide data on the distribution of paid rents for tenant-occupied houses. However, the RFS was the only one that provided data on the market value of these houses and its distribution. These data could be combined with paid rents on these houses to estimate rent-to-value ratios. This paper shows that imputed rents can be estimated without directly estimating such ratios.

In the absence of data on the distribution of market values of tenant-occupied houses, it is necessary to develop a method based on a statistical model of how imputed rents on owner-occupied houses relate to the rents actually paid on tenant-occupied ones. One could try to estimate the rental values of owned houses based on their characteristics. However, it is difficult to determine which characteristics are truly important and data on the most important characteristics, the unit’s location, will probably be missing.

This paper develops a methodology that directly models the distribution of imputed rents. The methodology is simple and transparent. It is based on factoring up the distribution of paid rents for single-family houses to account for the fact that the median owner-occupied house is more valuable than the median tenant-occupied house. The mechanics of making the estimates are straightforward. Standard interpolation techniques are used to determine the rents at two points on the actual distribution of rents paid on single-family houses. These values are then used to determine the rents at
various points on a gamma distribution obtained using a standard program. An adjustment is added in for a possible undercount for top-coded units at the high-end of the distribution. An added bonus is that a shortcut methodology that is a simple weighted average of the two rents used to fit the gamma distribution replicates the results of the unabridged methodology with amazing precision. Note that following current BEA methodology, mobile (manufactured) houses are excluded from this methodology and handled separately. The rents in the calculations are contract rents and not gross rents, which include all utilities.

This model-based methodology rests on a few key assumptions. First, it is assumed that there is a monotonic relationship between a house’s market value and the amount it can be rented for. In other words, the higher a house’s market value, the higher the rent that it can command. This implies that if a house is at the $i^{th}$ percentile of the distribution of market values, it will also be at the $i^{th}$ percentile of the distribution of rents. Second, it is further assumed that the $50^{th}$ percentile of the distribution of imputed rents will be at the 71.4 percentile of the distribution of rents paid on single-family houses and that the $90^{th}$ percentile of the distribution of imputed rents will be at the 94.2 percentile of the distribution of rents paid on single-family houses. Third, it is further assumed that the frequency distribution of imputed rents follows a displaced gamma function with an alpha of 3.5 and a beta of 1. Fourth, it is assumed that a possible undercount due to top-coding at the high-end tail of the distribution of imputed rents is captured by a judgmental adjustment explained in the text. The paper shows why all of these assumptions are reasonable.

The model-based methodology accurately models the underlying data. When fit with the gamma distribution using an alpha of 3.5, the fitted distribution looks like the “actual” distribution of imputed rents in 2011. Its mean is only 0.9 percent greater than the estimated actual mean when the special adjustment for top-coding is not made and it is 2.2 percent greater when this adjustment is made.

The model-based methodology yields estimates that are lower than those presently published by BEA. In the model-based estimates for 2011, the mean imputed rent for owner-occupied units is 31.6 percent higher than the actual mean rent for tenant-occupied single-family houses and 45.6 percent higher than the mean contract rent for all tenant-occupied units. These figures are similar to historical relative values of contract rents by type of dwelling and by size of dwelling unit found in the ACS and the COH, and they are consistent with relative gross rents by type of dwelling unit and by size of dwelling found in the AHS and the COH.
REFERENCES


Appendix I. – BEA’s current methodology

BEA’s NIPA handbook (2012, chapter 5, p. 42) describes how data from the rent-to-value schedule was used to estimate the mean contract rent for the survey year, which was used as a benchmark. First, a unit-weighted average rent-to-value ratio was estimated for each market-value class of one-unit tenant-occupied dwellings. Then, the ratio for each value class was multiplied by the midpoint housing value for that class to derive an average rent per unit for the class. Next, the average rent per unit for each value class was multiplied by the corresponding number of owner-occupied units to derive the imputed rent receipts for these units - these weights were taken from the corresponding value classes of the AHS.\(^\text{13}\) The rent receipts and number of owner-occupied units were summed across all value classes and then the former was divided by the latter to derive an imputed mean rent for owner-occupied permanent-site houses.

Non-benchmark annual estimates of owner-occupied contract rent per unit are estimated by extrapolation using the product of (1) the CPI for owners’ equivalent rent, which captures changes in the rental value of constant-quality owner-occupied dwelling units, and (2) the constant-dollar per-unit value of owner-occupied nonfarm dwelling units, which captures changes in the rental value that result from changes in the average quality of these dwelling units.

Note, that the above methodology is applied only to nonfarm permanent site houses. The space rents of farm housing and mobile homes are estimated separately. BEA also imputes space rent to only a relatively small number of second houses, those that are held for the occasional use of their owner. It does not impute rent to other categories of second houses, primarily vacation homes.

Appendix II. – Fitting the gamma distribution

The gamma distribution was fit using a standard program for the gamma distribution obtained over the Internet. Beta is set equal to 1 and the desired values of alpha and Q (the number of equally spaced observations over the distribution of rents) are also chosen. \( Q \) is set equal to a number high enough so that the highest cumulative probability shown on the outputted table is at least 99.8 percent.

Thus, if \( Q \) is set equal to 100, the program produces a table that gives for each of 100 equally spaced values, the probability that the variable of interest takes on a value less than the value at that point. This can be interpreted as saying that if that point is associated with a given rent, call it \( r_1 \), the distribution gives the percent of dwelling units with a rent lower than \( r_1 \). At the next lower point, we similarly know the percent of dwelling units with a rent lower than \( r_0 \), the rent associated with that point. By subtraction, we can calculate the percent of dwelling units with a rent between \( r_0 \) and \( r_1 \). Because the difference between the rents in successive points is a constant, we can completely determine the distribution by knowing the rent at the first observation, the 0 percentile, and the constant difference in rents between successive steps.

Now, let \( Z = \) the lowest value on the fitted distribution of rents (the rent at the 0 percentile).

\(^{13}\) Prior to 1984, the survey was termed the Annual Housing Survey. After 1981, the survey was conducted only in odd numbered years and renamed the American Housing Survey.
R50 = the rent at the 50th percentile of rents (as determined by observation or assumption)
R90 = the rent at the 90th percentile of rent (as determined by observation or assumption)
F = the difference between the rents associated with adjacent observations.
Q50 = the number of the observation (estimated by interpolation using the output for the
gamma distribution schedule) associated with the 50th percentile of the distribution.
Q90 = the number of the observation (estimated by interpolation using the output for the
gamma distribution schedule) associated with the 90th percentile of the distribution.

By definition, we have R50 = Z + F \cdot Q50 and R90 = Z + F \cdot Q90

Subtracting the 1st equation from the second and solving for F, we obtain

\[ F = (R90 - R50) / (Q90 - Q50) \]

Knowing F, we can then solve for Z.

Knowing F and Z, we can then compute the rents at each of the Q observations. The output of the
program gives us the percentile associated with each of these observations. Note that the methodology
described here can be used to fit curves for any distribution, not just the gamma distribution.

Thus, the numbers for 2011 were fit as follows. From the 2011 ACS it was determined that R50,
which is at the 71.4 percentile of the distribution of paid rents) is 1,071.79 and that R90, which is at the
94.2 percentile of the distribution of paid rents, is 1,908.48. Then, alpha was set equal to 3.5 and Q was
set equal to 113 and the program outputted a table giving the cumulative probability distribution. From
this table, it was determined that Q50 occurs at the 31.73 observation while Q90 occurs at the 60.09
observation. Substituting these numbers into the equation given above we determine that

\[ F = (1,908.48 - 1,071.79) / (60.09 - 31.73) = 29.51 \]

Then Z is solved for by calculating that Z = R50 – (F \cdot Q50) = 1,071.79 – (29.51 \cdot 31.73) = 135.51.

Appendix III. – Using ACS data

The annual 1-year estimates from the ACS are a source of data on contract and gross rents. Using theACS offers a number of compelling advantages. First, it will permit BEA to continue its long-established procedure of “benchmarking” housing data. While alternative estimates of the value of a
given variable may be available from other surveys or sources, BEA has adhered to the policy of
selecting only one source as representing the “truth.” In the past, a detailed Census of Housing (COH)
was conducted in conjunction with the decennial Census of Population using the long form. Data from
this census was regarded as the truth and used to benchmark all housing related data. The long form
was abandoned after 2000. The Census Bureau developed the ACS as a replacement for it.

The ACS has a very large sample size. In recent years, observations have been reported for over
2 million households that have been drawn from about 3 million addresses. (About 40 percent of these
observations are included in the public use sample.) Its principal competitors - the AHS, the Current Population Survey (CPS), and the Consumer Expenditure Survey (CE) - are much smaller. The CPS is drawn from about 72,000 addresses while the AHS is drawn from about 62,000 and the CE interviews only about 21,000 households per year. The ACS is a mandatory survey while the other surveys are voluntary.

For BEA’s purposes, the ACS is superior to its competitors in other ways. The sample design of the AHS is problematical. Most units in the AHS sample remain there for many years, if not decades. This helps in measuring short-term changes for given units, but makes the sample unrepresentative of the overall stock. The sample was completely redrawn in 2015 and estimates from the new survey (and subsequent ones) cannot be directly compared with data from the old ones. In other words, there is a major series break. Also, data on certain variables taken from the CPS and AHS, such as the number of second houses, differ more from the decennial Census estimates than do the estimates from the ACS.

A major advantage of using the ACS is that data is available annually. In the past, BEA interpolated housing data between benchmark years using data from the AHS and other annual surveys. However, the AHS is available only every second year. BEA used it to measure growth rates in odd-numbered years and used data from other surveys in even-numbered years. The availability of annual data in the ACS makes this interpolation unnecessary. This not only saves time, it reduces the size of errors and revisions that occur because of the interpolation process.

Another advantage of using the ACS is that a public use micro-data sample from it is now available using the Census Bureau’s “FERRET” software. Data from this sample is available approximately 18 months after the end of the calendar year. The software enables the user to cross-tabulate data using multiple variables. For example, it was used to estimate a distribution of contract rents for single-family houses that were rented out and could exclude mobile homes and vacant units. In fact, the software enabled separate distributions for attached and detached units to be estimated, although the two were aggregated in the methodology used here. The software also enables the user to recode variables and break continuous variables down into narrow intervals. Even the special tabulations of the RFS that BEA has received in the past had their values presented in very wide intervals. These wide intervals caused problems at the high end of the distributions.

In conjunction with the FERRET software, the ACS provides detailed data that is simply not available elsewhere. Data on the difference between gross and contract rents by type of building and number of bedrooms is not available in published tables. In fact, data on the distribution of contract rent by size of rent classes is also not available in published tables. However the FERRET software enables one to break contract rent down into intervals as small as $2 using the recoding features of the software.